Semantics II

Andrew Murphy
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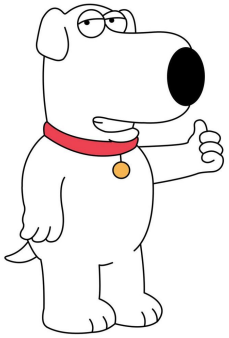
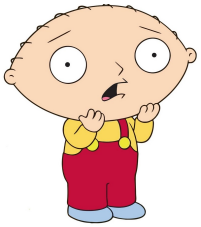
Week 8
05.18.22
LING 20001: Introduction to Linguistics

Summary

Linguistic expressions divide up the logical space of entities in the world:





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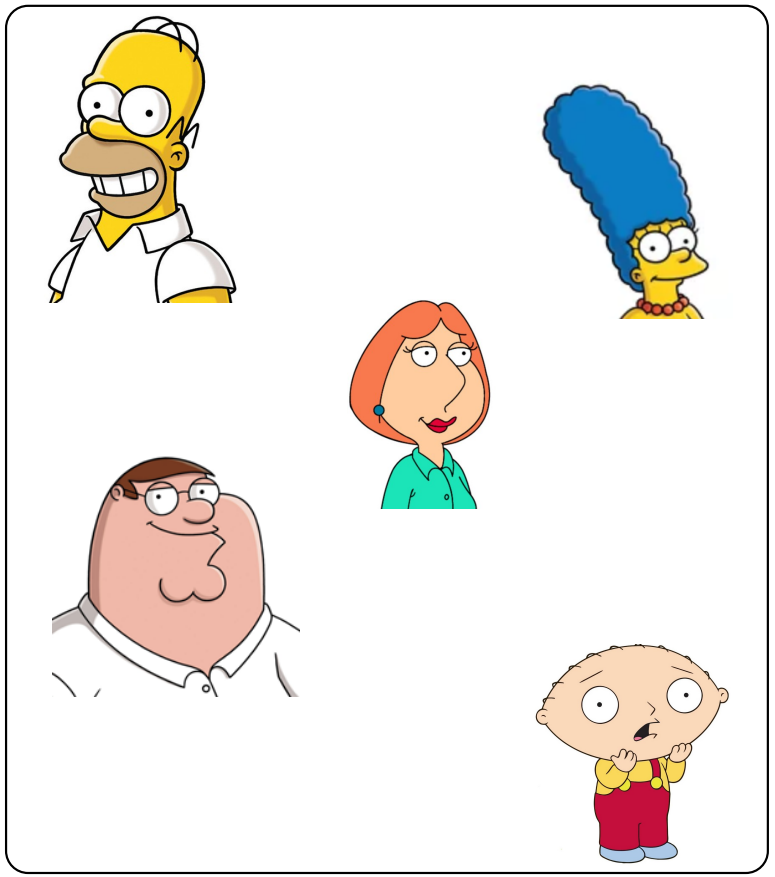

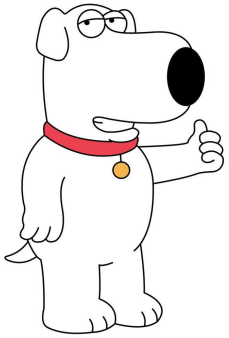

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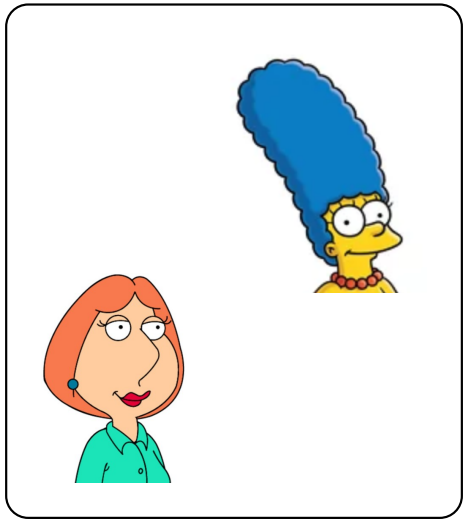

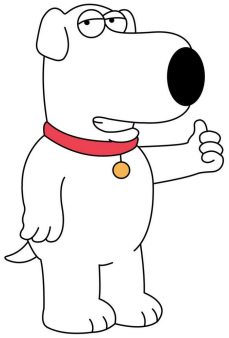

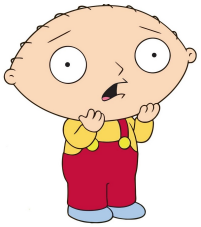

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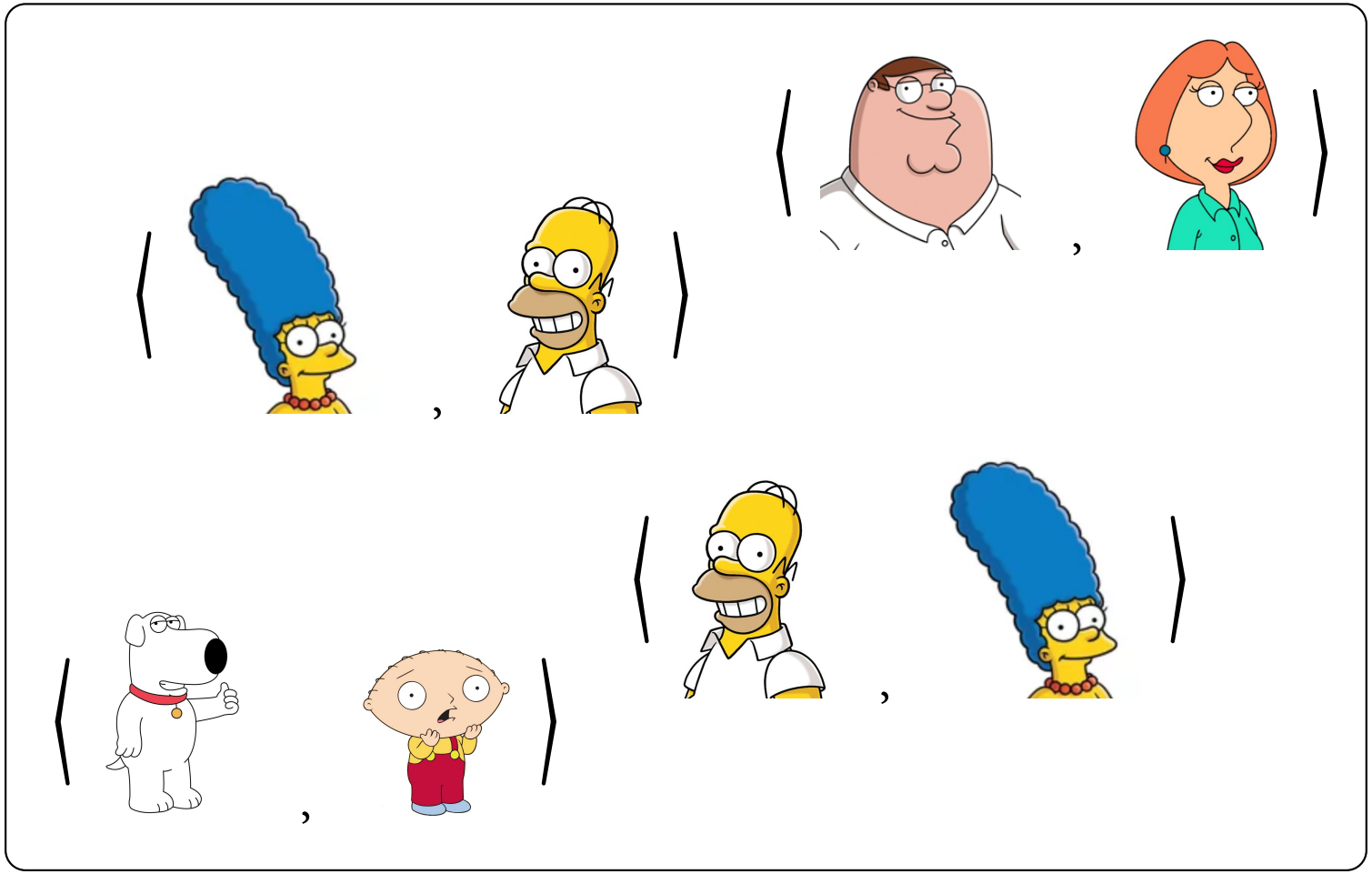

$[\![\text{love}]\!]$

Summary

Two interpretation rules, one for intransitive structures, and one for transitives:

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$$\left[\!\!\left[\begin{array}{c} \text{S} \\ \text{NP}_1 \quad \text{VP} \\ \text{N} \quad \text{V} \quad \text{NP}_2 \\ \text{Lois} \quad \text{loves} \quad \text{N} \\ \text{Peter} \end{array} \right]\!\!\right] = \begin{array}{l} \text{true if } \langle [\![\text{NP}_1]\!], [\![\text{NP}_2]\!] \rangle \in [\![\text{V}]\!] \\ \text{true if } \langle \text{Lois, Peter} \rangle \in \{\langle x, y \rangle : x \text{ loves } y\} \end{array}$$

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What is the meaning of *Lois loves Peter*?

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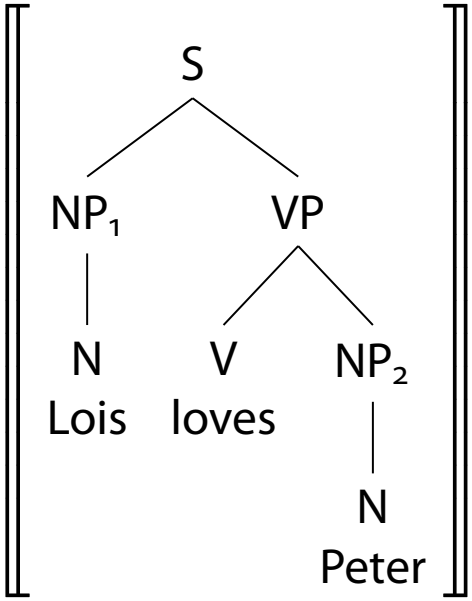

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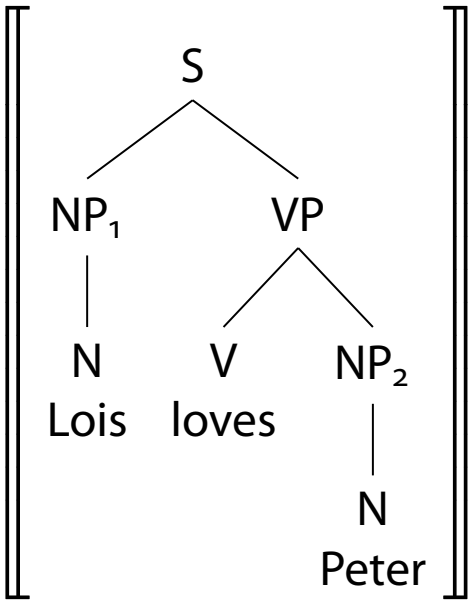

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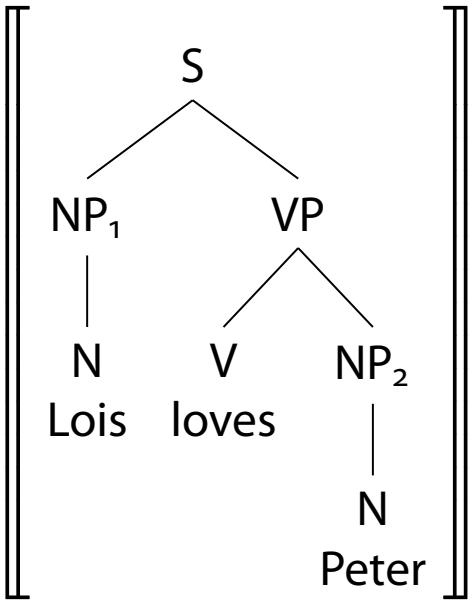

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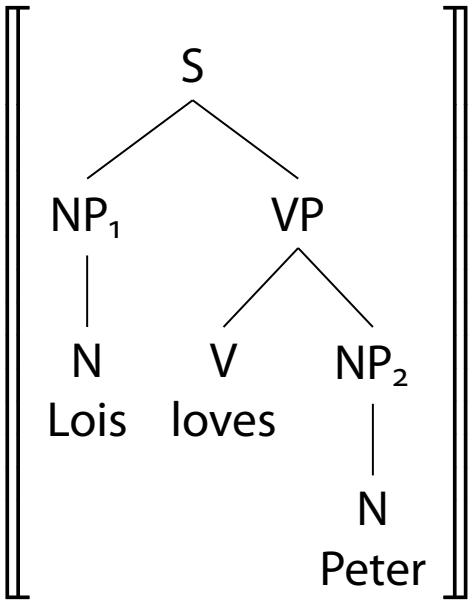

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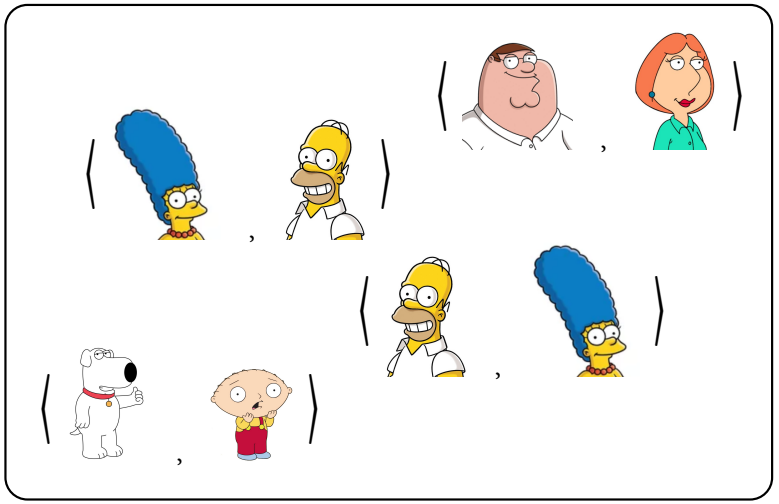

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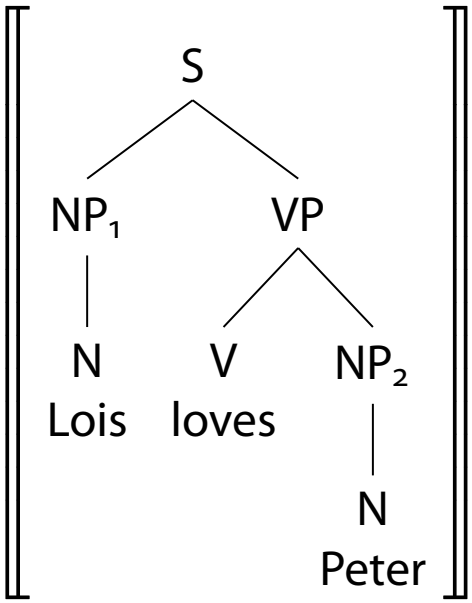

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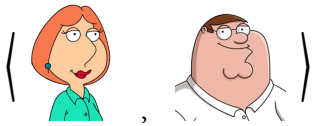
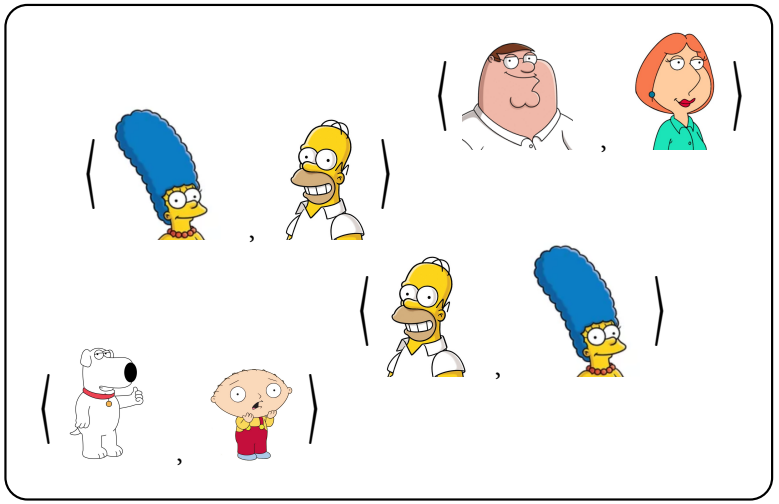
true if  $\in$ 

false

The meaning of NP

What about a sentence like *The train has arrived*?

$$\left[\!\left[\begin{array}{c} \text{NP} \\ \text{Det} \quad \text{NP} \\ \text{the} \end{array} \right]\!\right] = \text{the unique } x \text{ s.t. } x \in [\![\text{NP}]\!]$$

What about other determiners like *every*?

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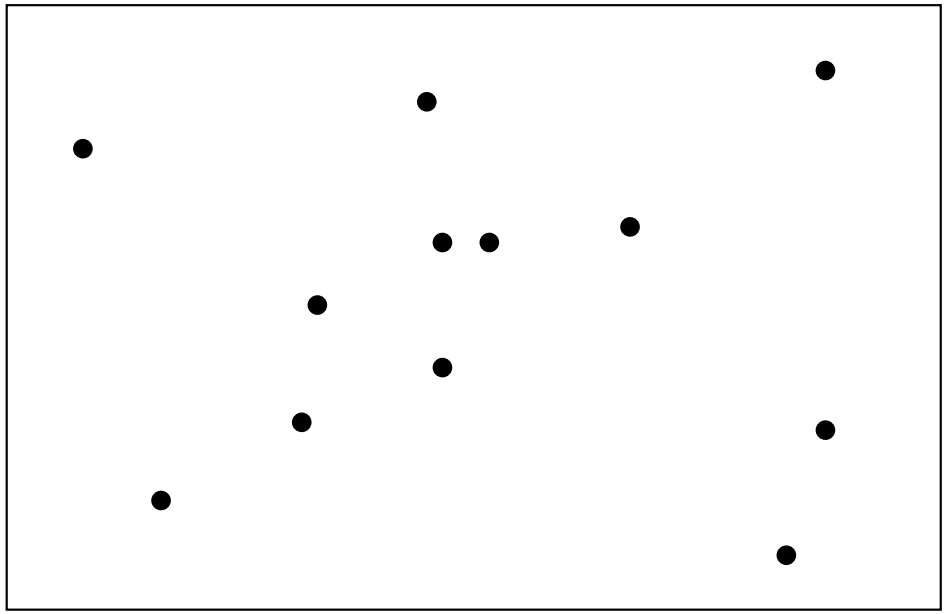

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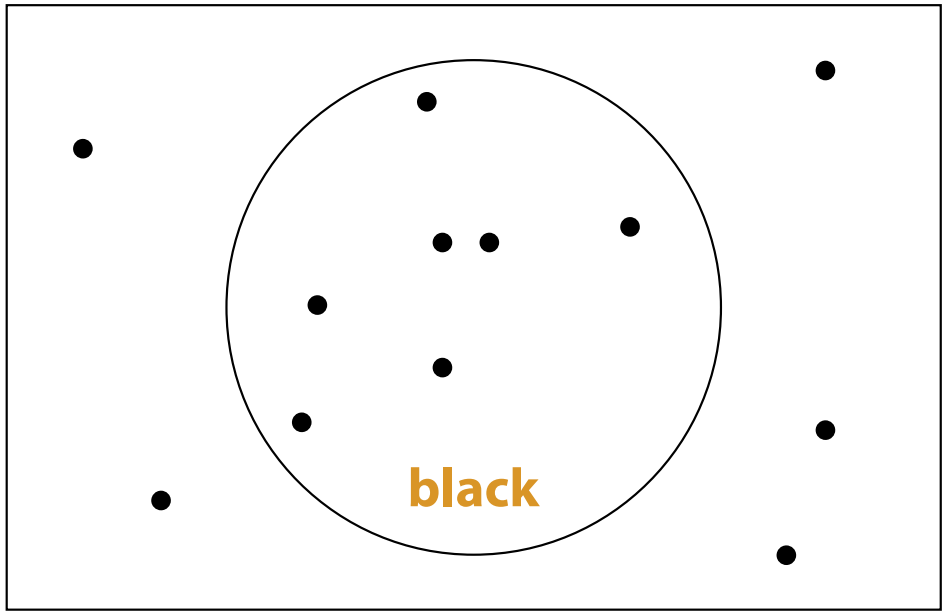

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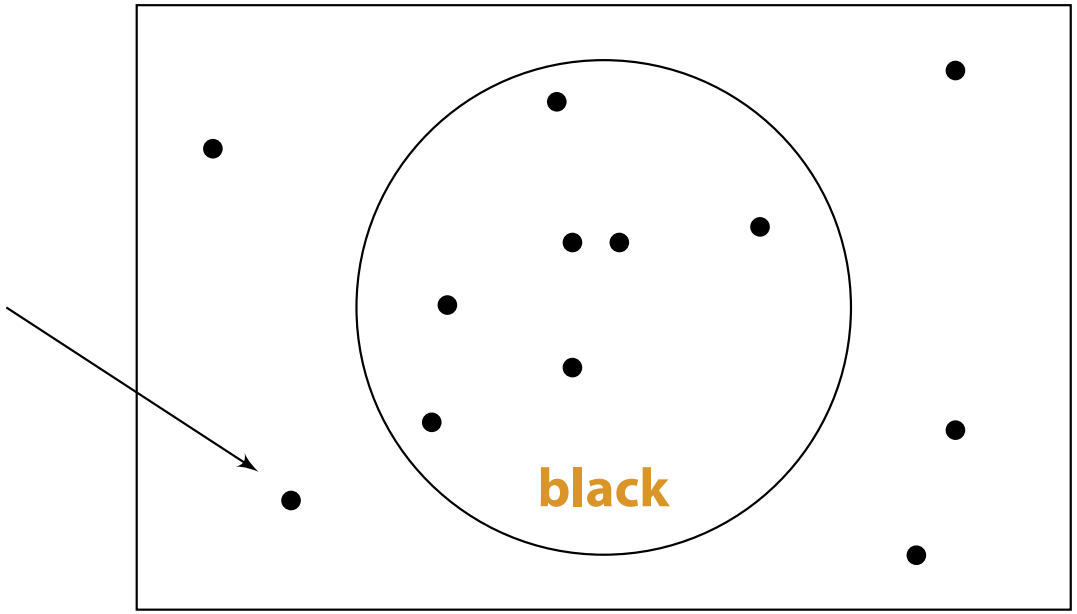

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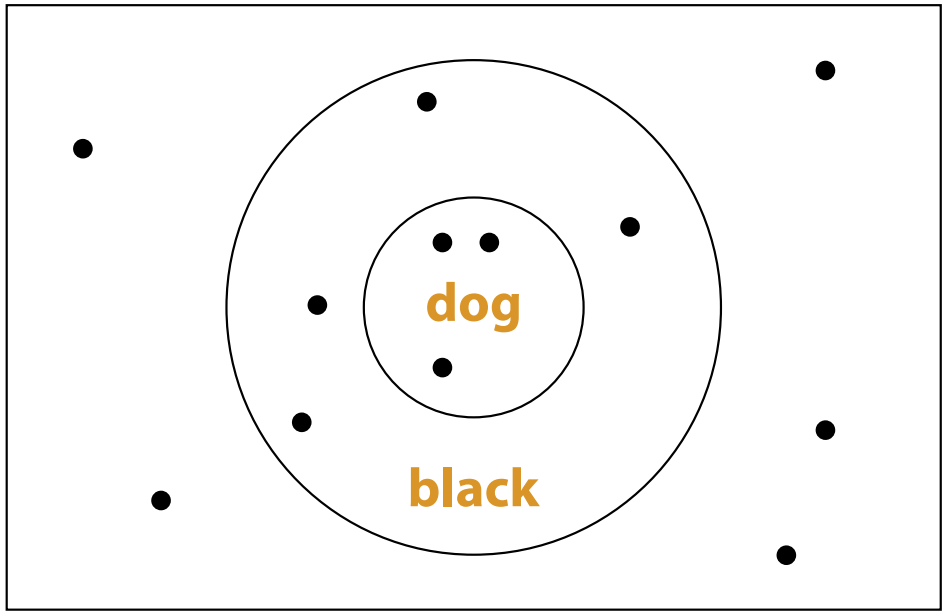

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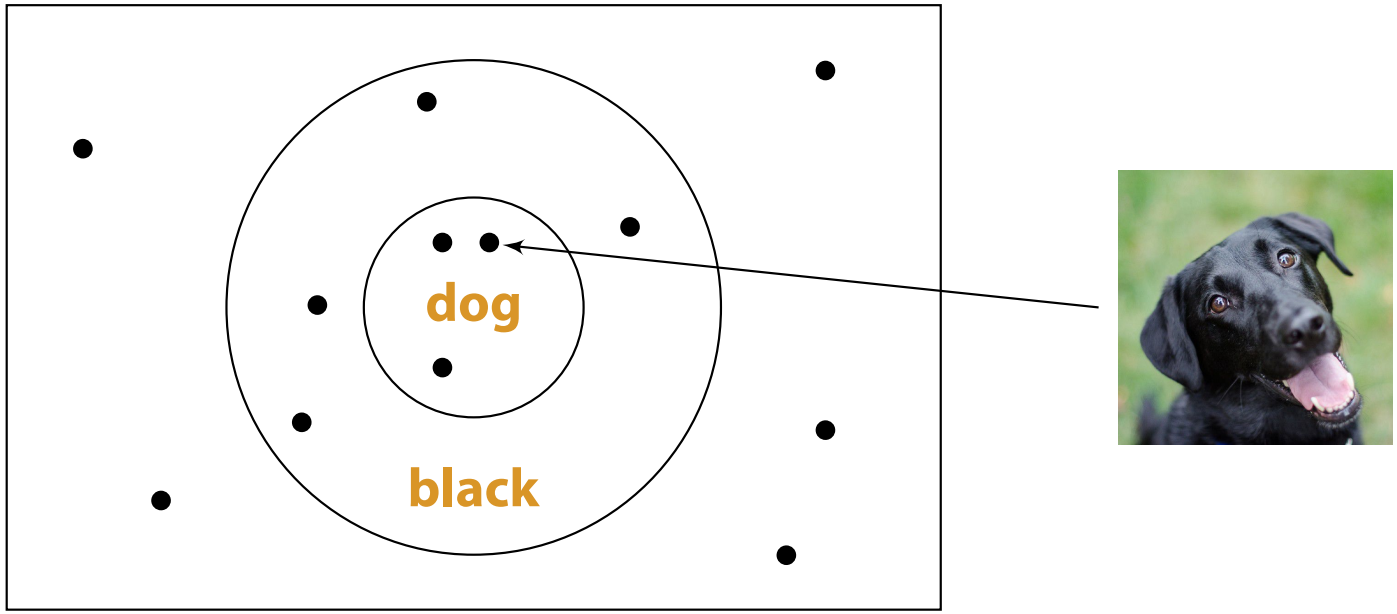

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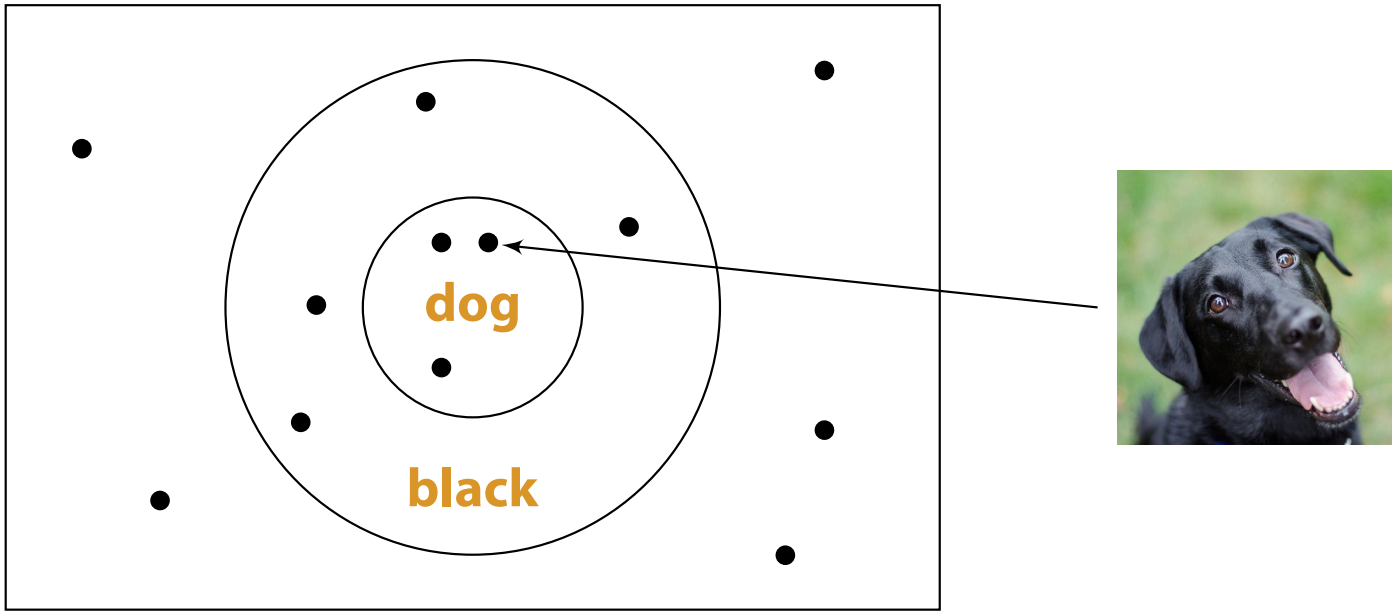

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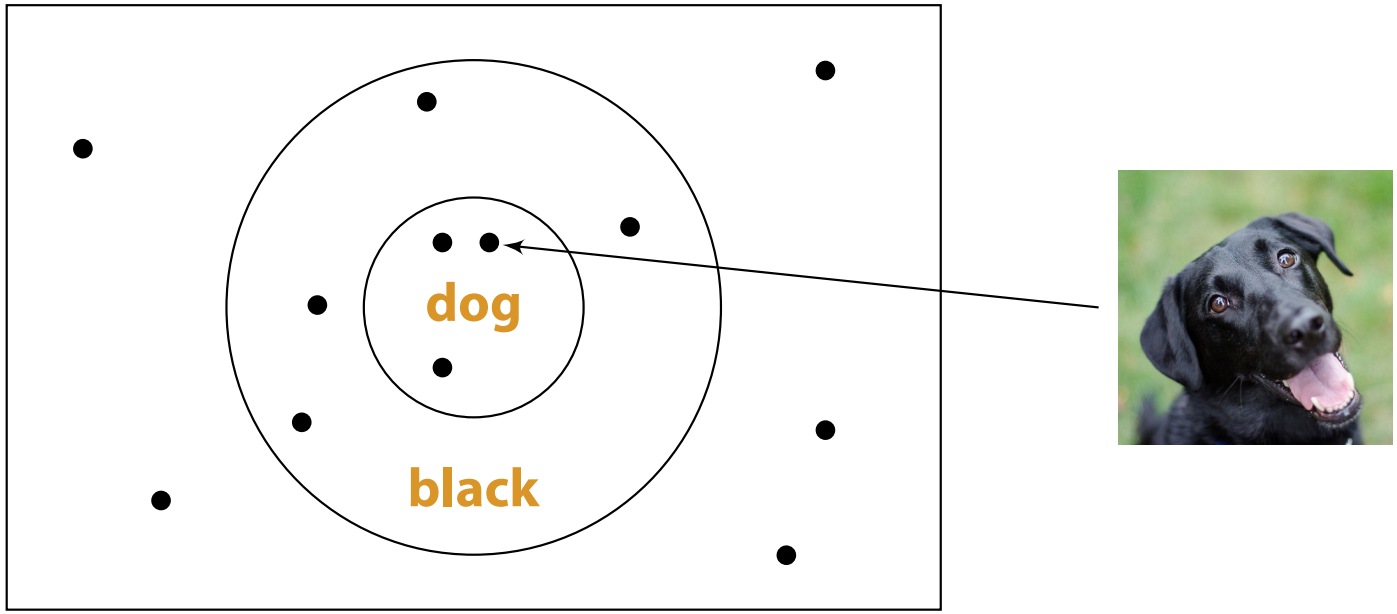

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$$[\![\text{Every dog is black}]\!] = \{x : x \text{ is a dog}\} \subseteq \{y : y \text{ is black}\}$$

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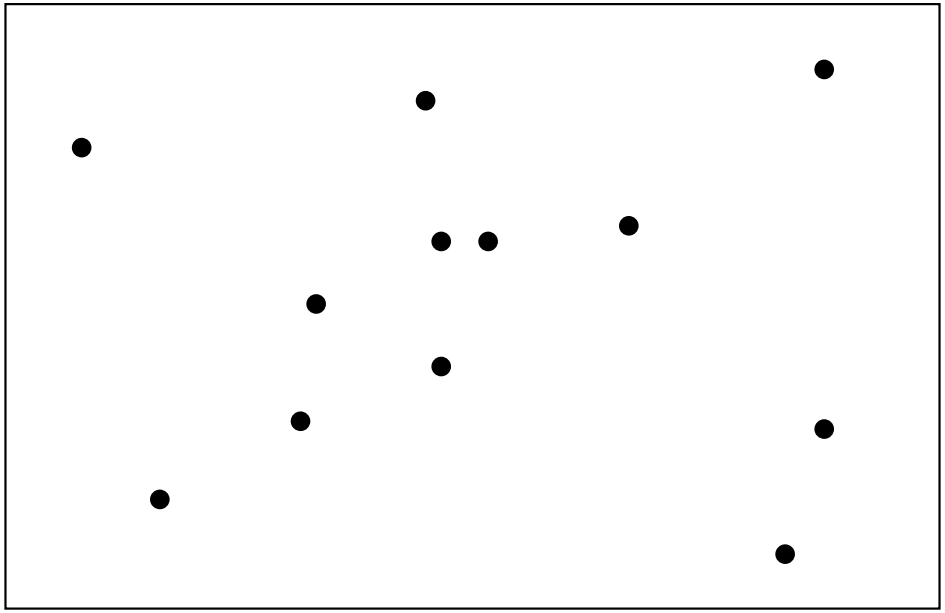

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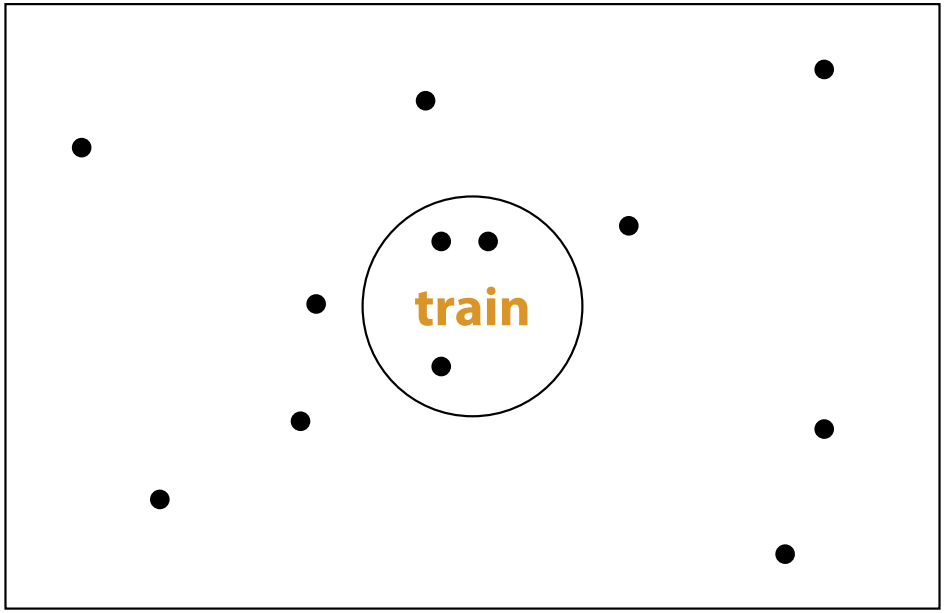

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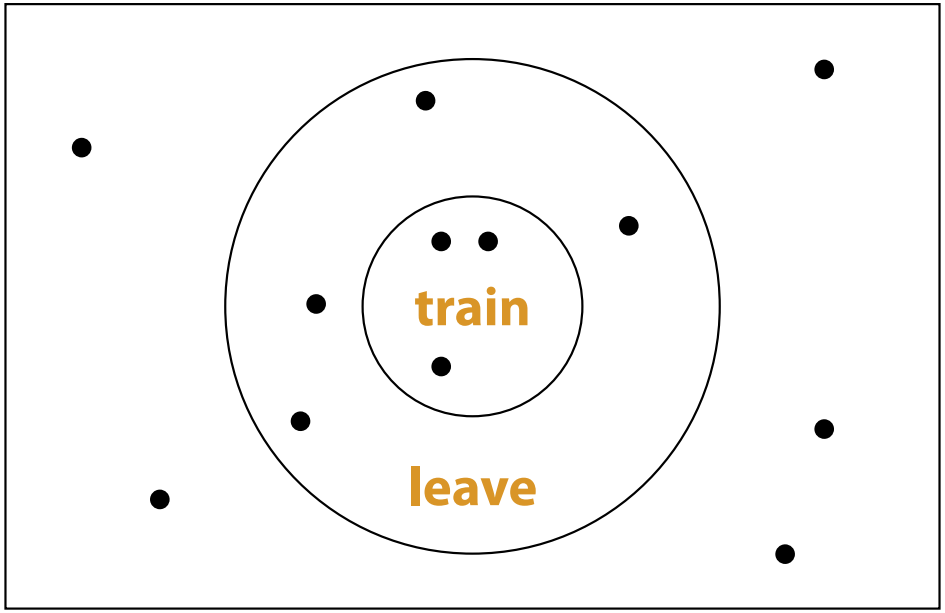

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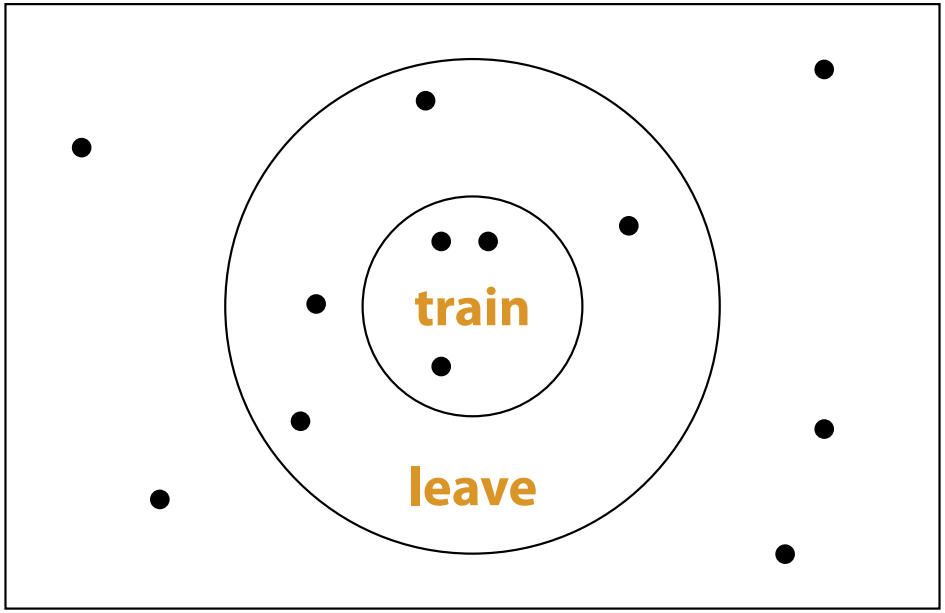

$$[\![\text{Every train left}]\!] = \text{true if } \{x : x \text{ is a train}\} \subseteq \{y : y \text{ left}\}$$

Compositionality

How can we arrive at a compositional rule for quantifiers?

The rule we have so far for intransitive verbs is this:

$$\left[\!\!\left[\begin{array}{c} \text{S} \\ \text{NP} \quad \text{VP} \\ \quad\quad\;\; | \\ \quad\quad\;\; \text{V} \end{array} \right]\!\!\right] = \text{true if } [\![\text{NP}]\!] \in [\![\text{V}]\!]$$

But this won't work…

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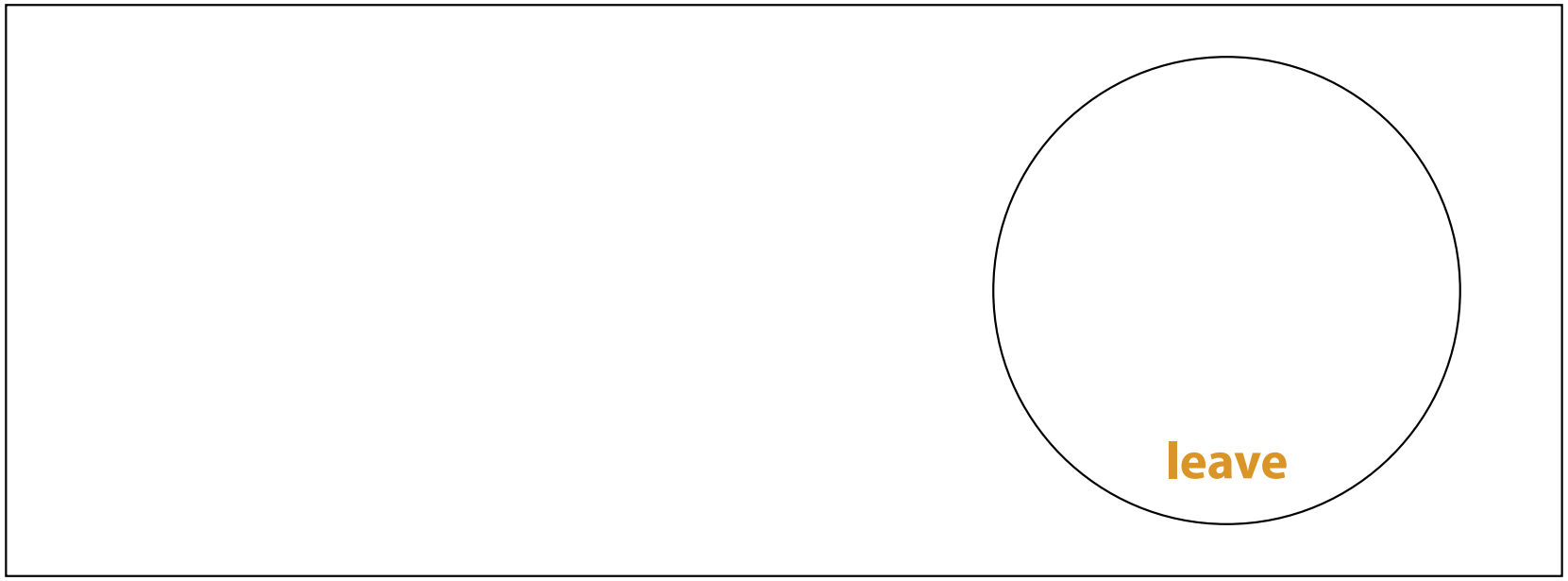

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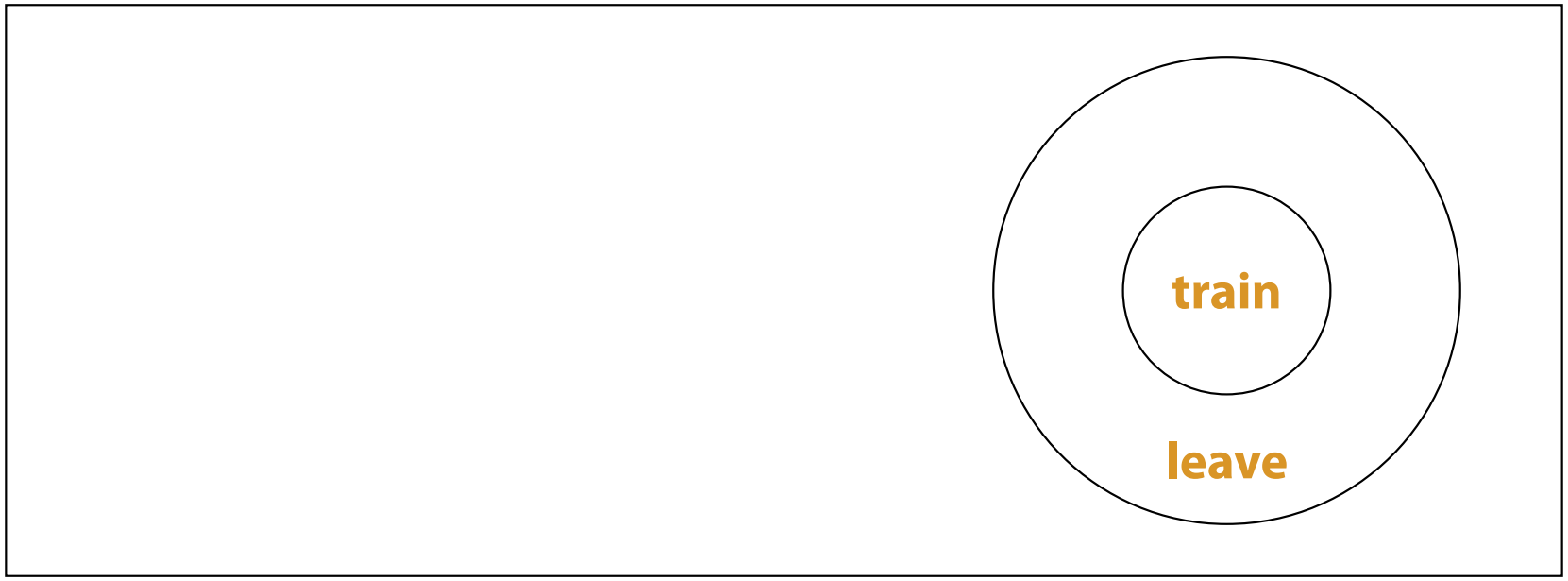

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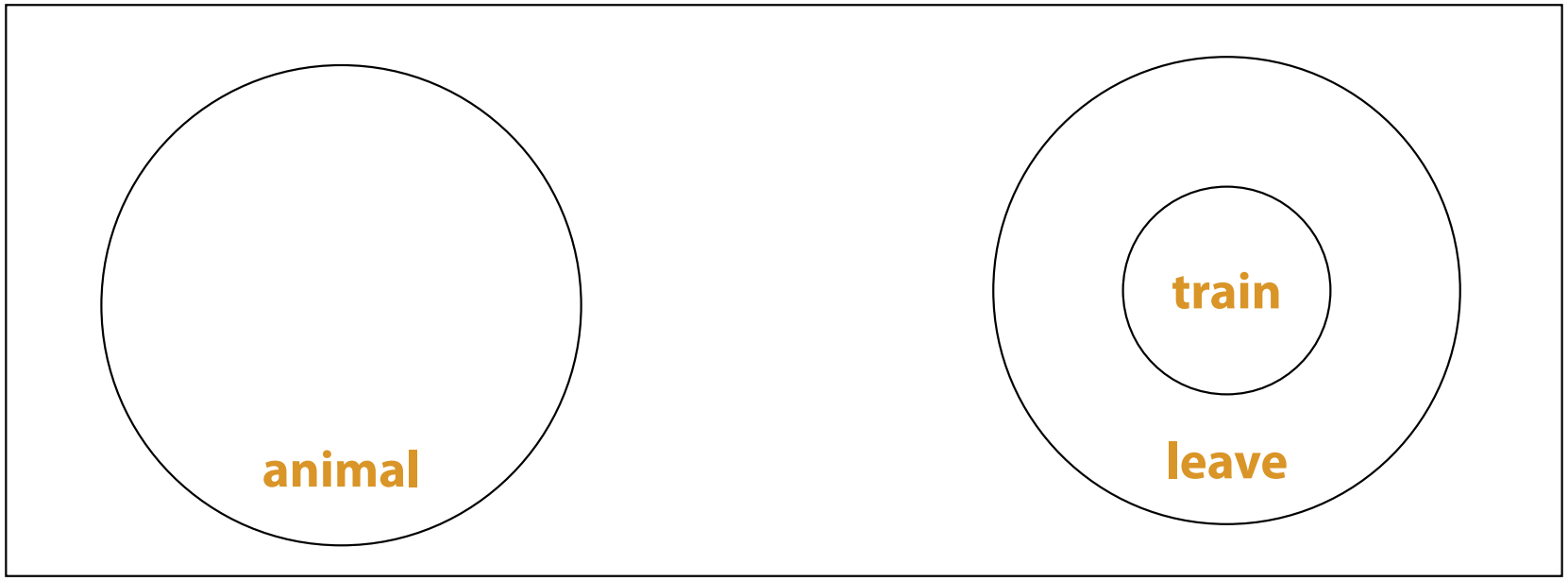

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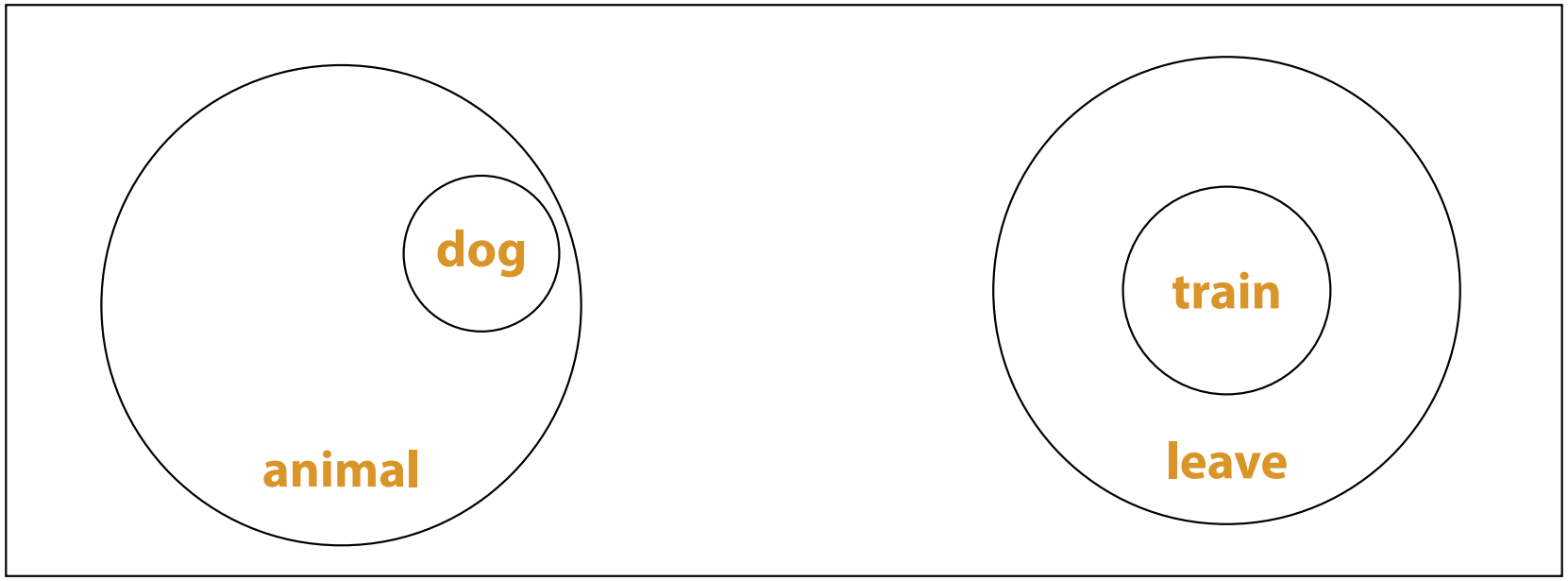

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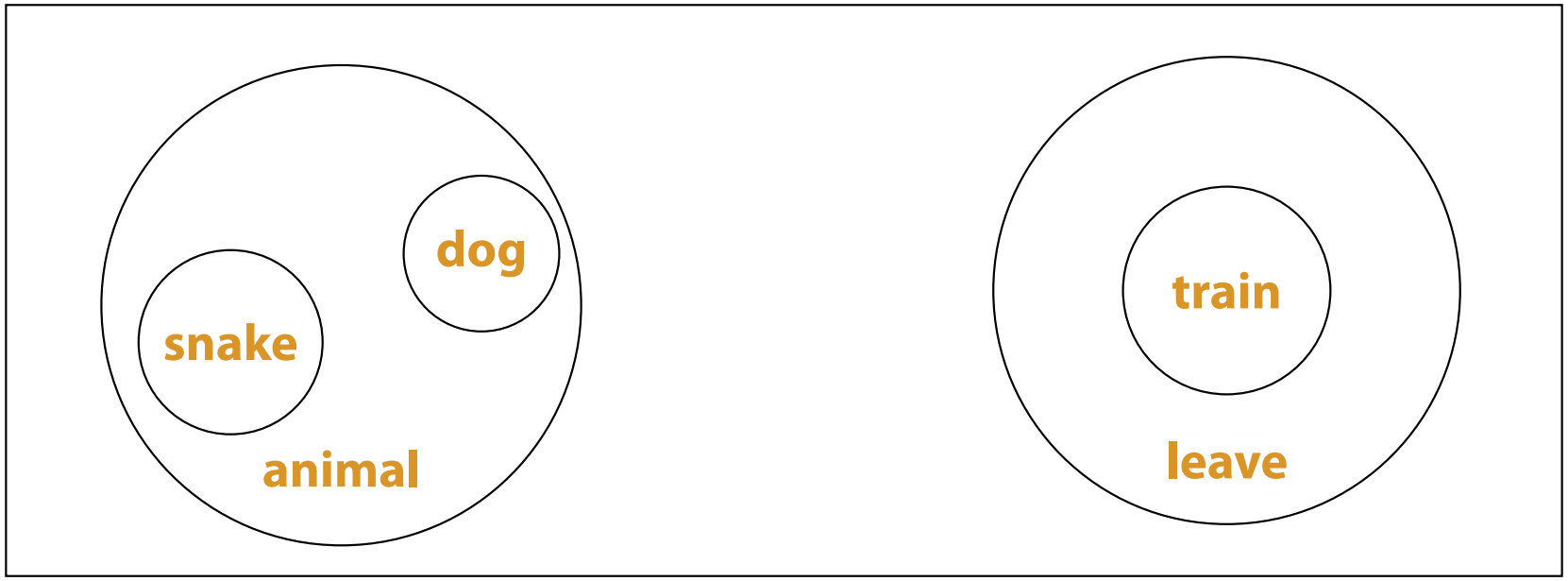

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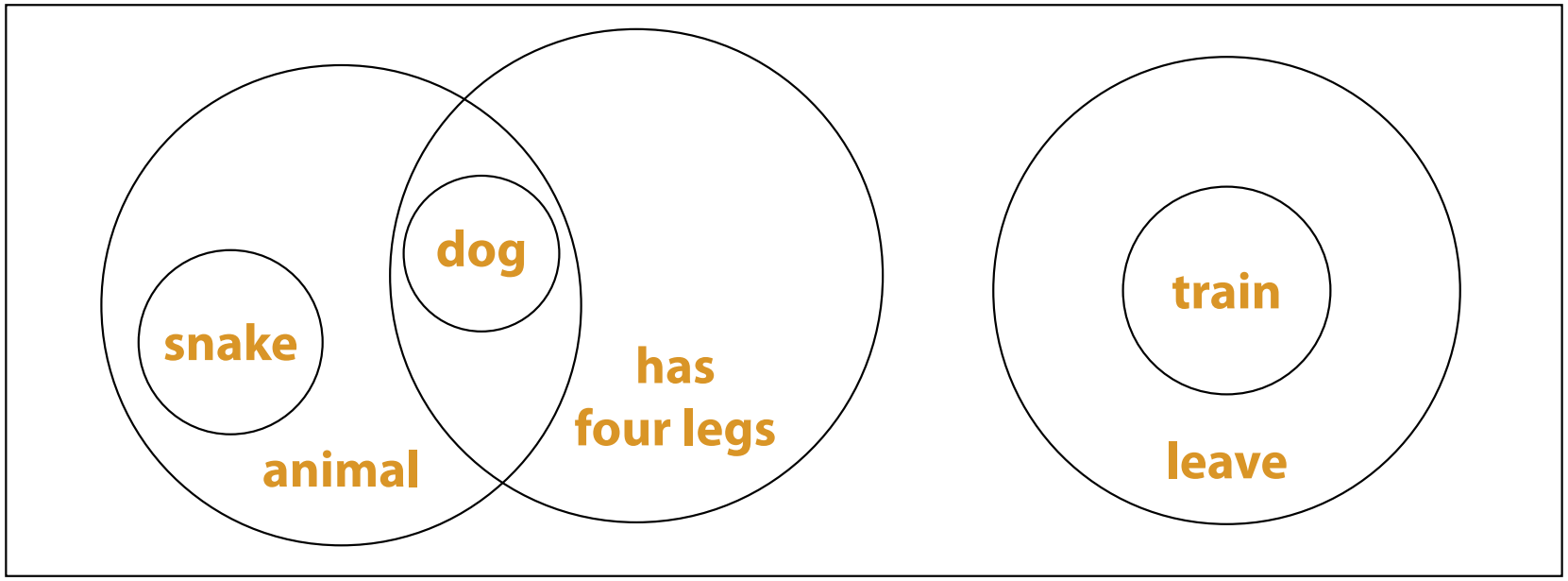

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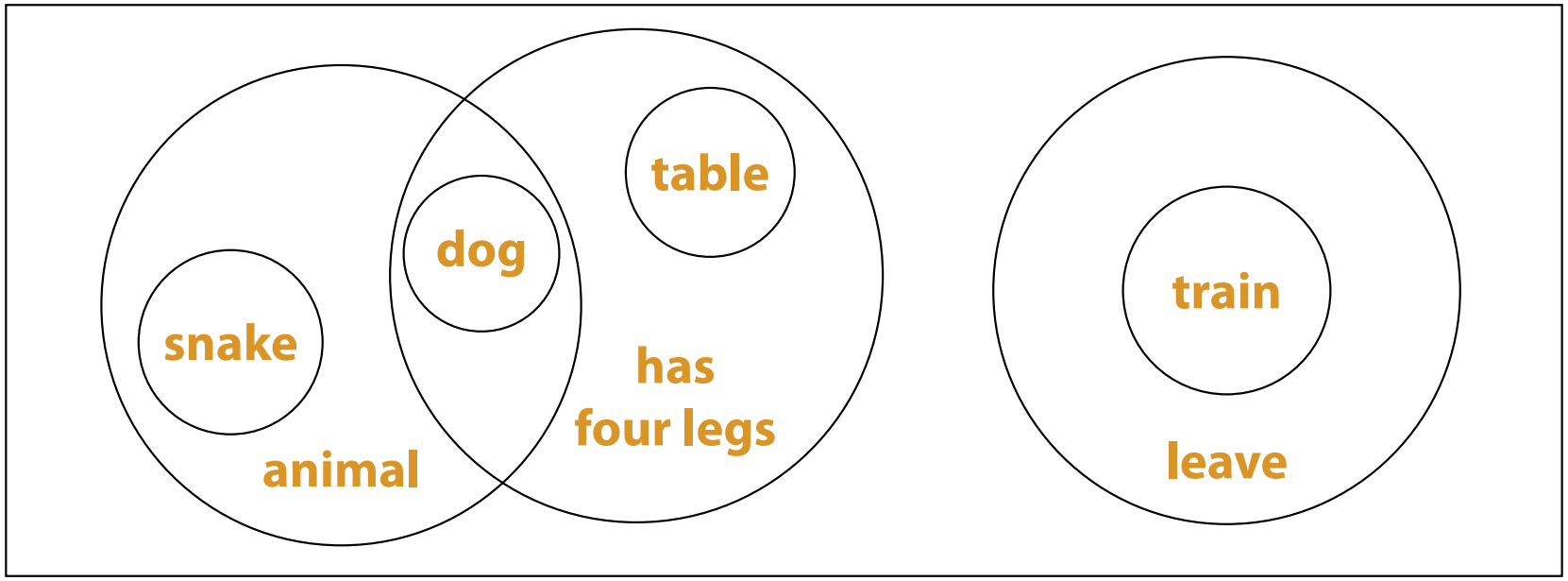

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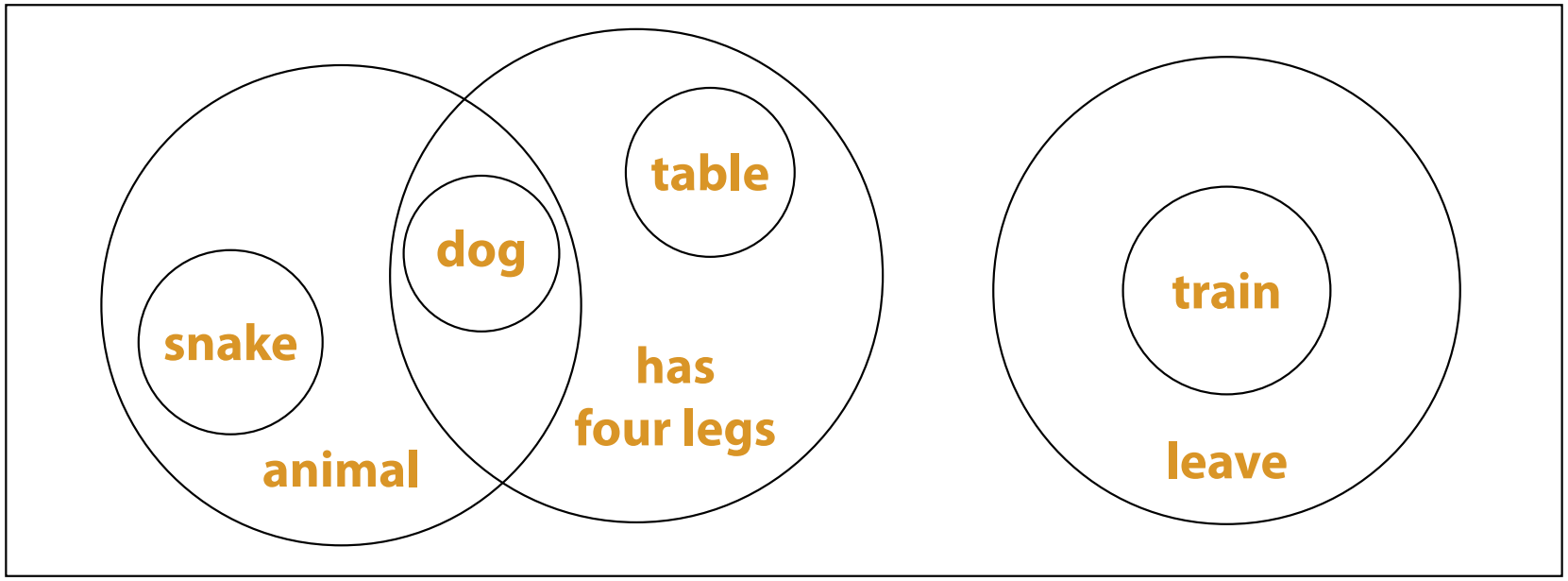

This relation can be expressed as a set of ordered pairs of **sets**:

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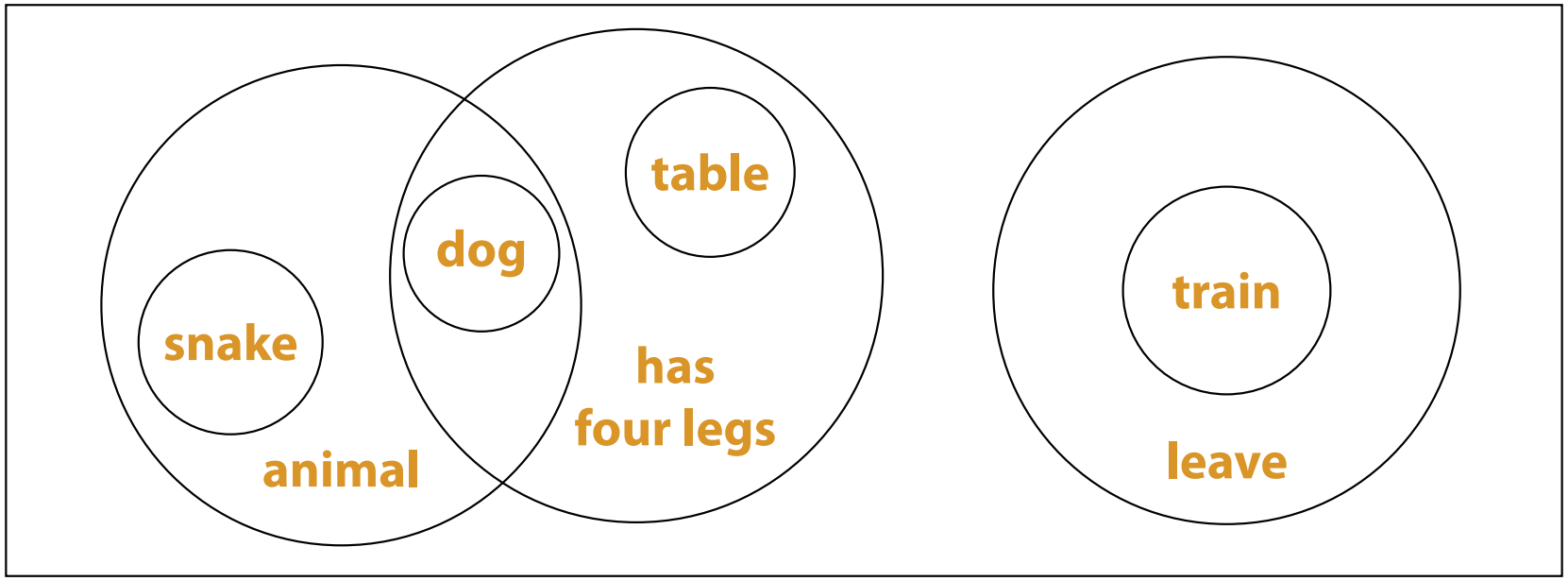

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$$\{\langle [\![\text{snake}]\!], [\![\text{animal}]\!]\rangle, \langle [\![\text{dog}]\!], [\![\text{animal}]\!]\rangle, \langle [\![\text{dog}]\!], [\![\text{have four legs}]\!]\rangle, \ldots\}$$

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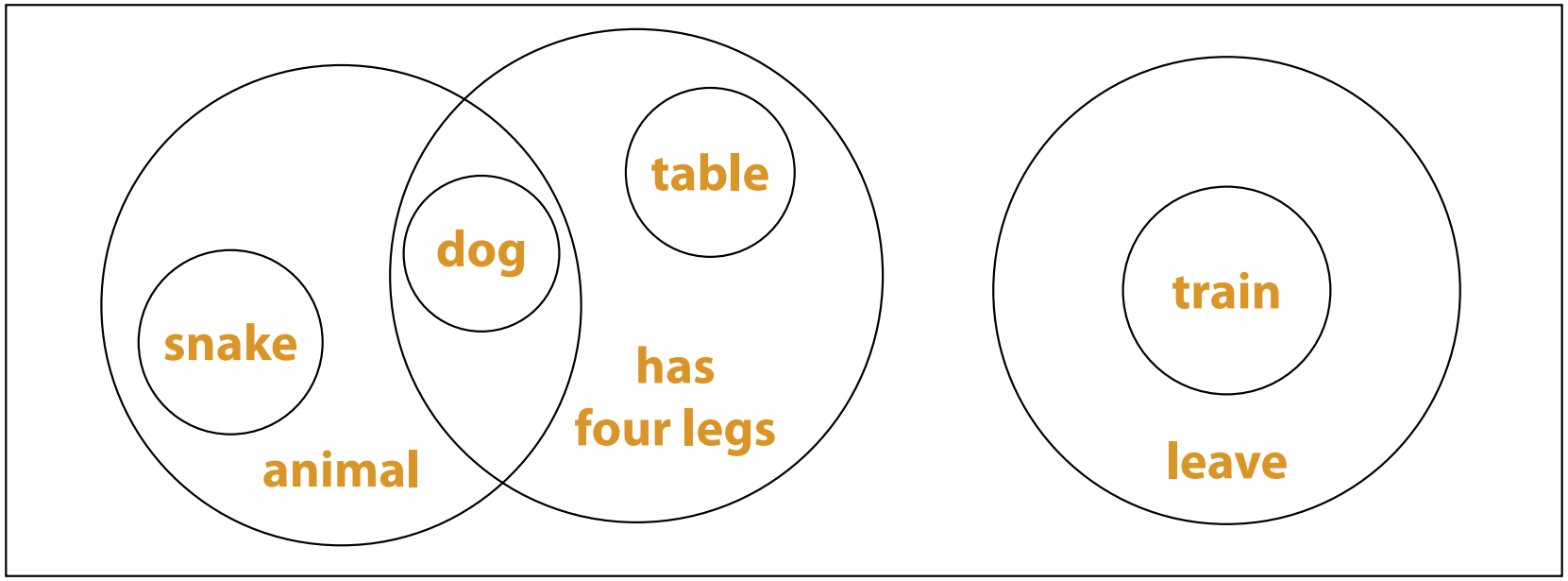

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Every always relates **hyponyms** (like *dog*) to their respective **hypernym** (*animal*)

Compositionality

$$\left[\!\left[\begin{array}{c} \text{S} \\ \text{NP} \quad \text{VP} \\ \text{Det} \quad \text{NP} \quad \text{V} \\ \text{every} \quad \text{N} \quad \text{arrived} \\ \text{train} \end{array} \right]\!\right] =$$

true if $\langle [\![\text{NP}]\!], [\![\text{VP}]\!] \rangle \in [\![\text{Det}]\!]$

true if $\langle [\![\text{train}]\!], [\![\text{arrived}]\!] \rangle \in [\![\text{every}]\!]$

true if $\langle \{x : x \text{ is a train}\}, \{y : y \text{ arrived}\} \rangle \in \{\langle \text{P}, \text{Q} \rangle : \text{P} \subseteq \text{Q}\}$

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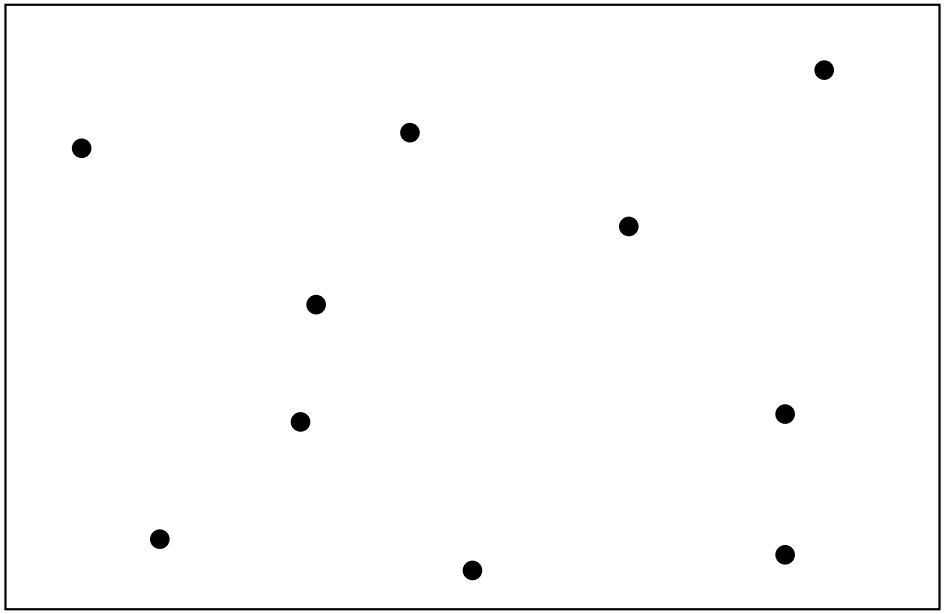

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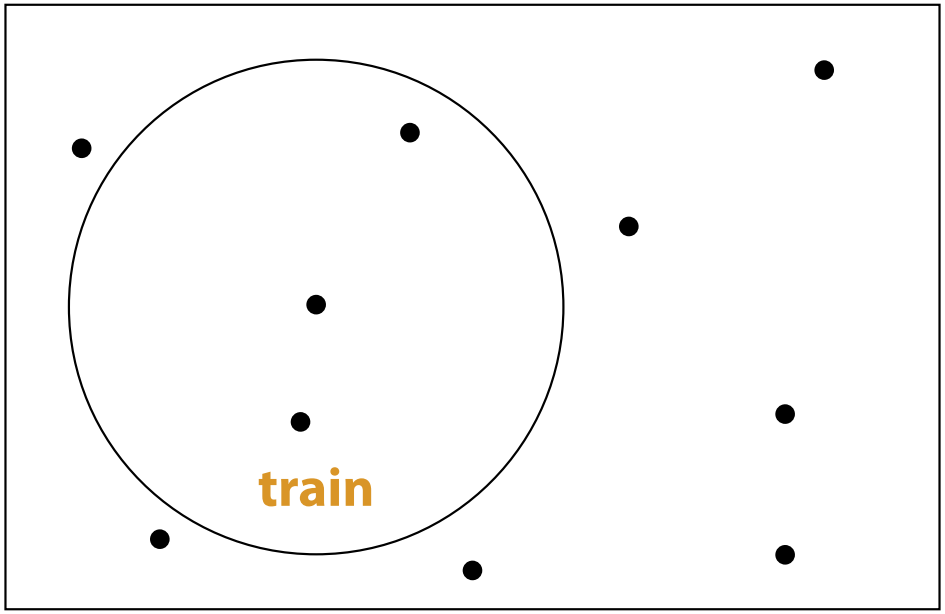

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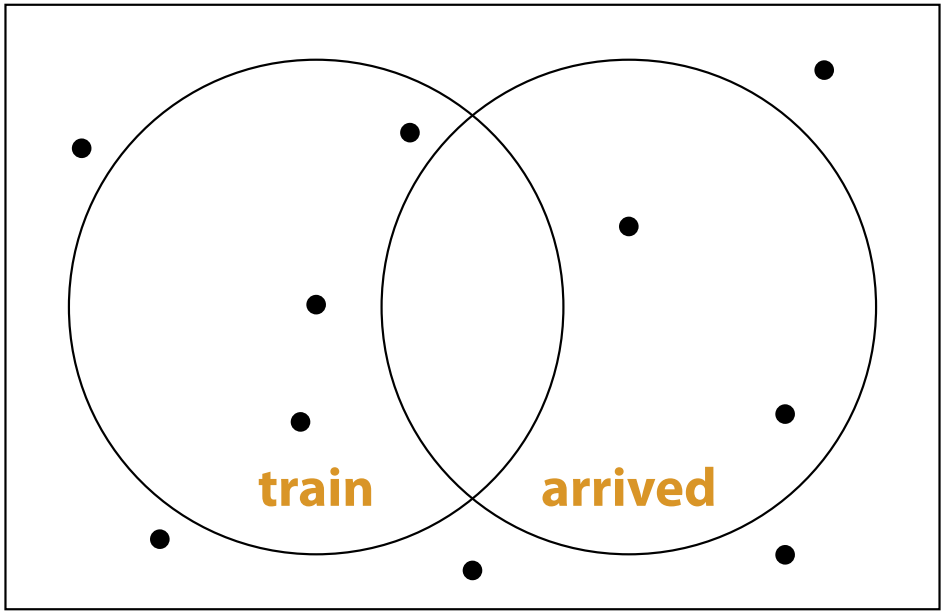

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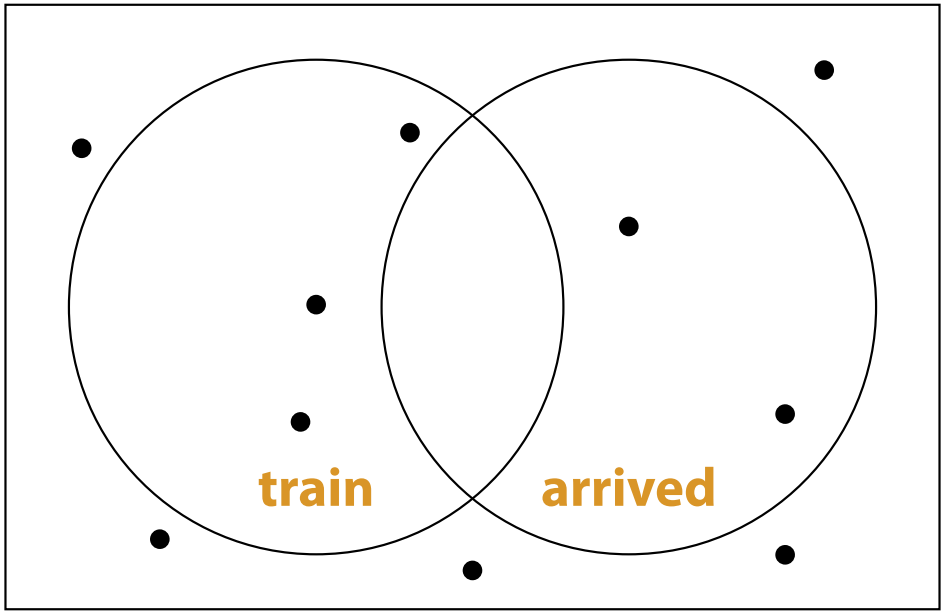

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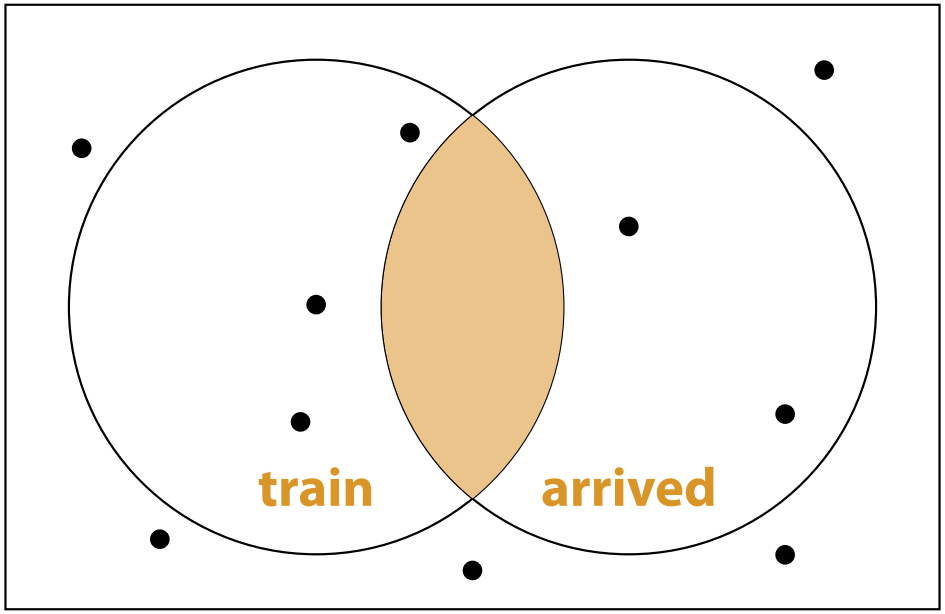

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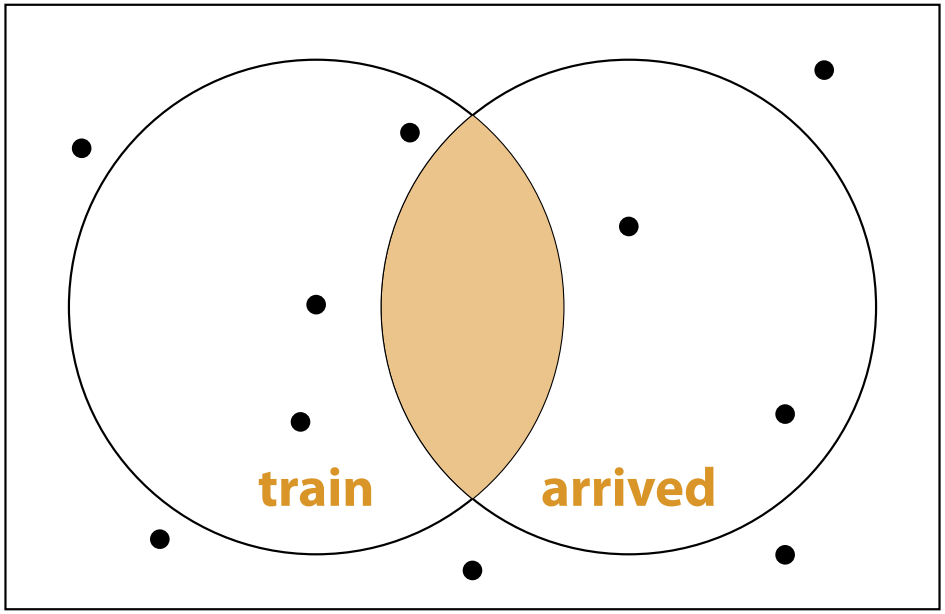
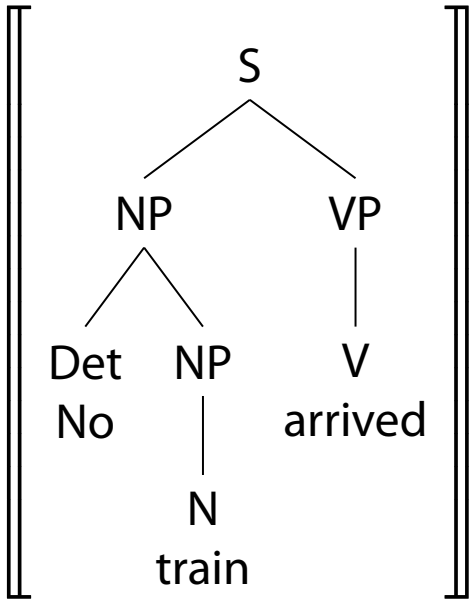


In set-theoretic terms: The **intersection** of the two sets ($\cap$) must not contain any elements, i.e. it must be **empty set** ($\emptyset$)

$$[\![\text{No train arrived}]\!] = \text{true if } \{x : x \text{ is a train}\} \cap \{y : y \text{ arrived}\} = \emptyset$$

Compositionality

How does this all fit together compositionally?



$$= \quad \text{true if } \langle [\![\text{NP}]\!], [\![\text{VP}]\!] \rangle \in [\![\text{Det}]\!]$$

$$\text{true if } \langle [\![\text{train}]\!], [\![\text{arrived}]\!] \rangle \in [\![\text{no}]\!]$$

$$\text{true if}$$

$$\langle \{x : x \text{ is a train}\}, \{y : y \text{ arrived}\} \rangle \in \{\langle \text{P}, \text{Q} \rangle : \text{P} \cap \text{Q} = \emptyset\}$$

$$[\![\text{no}]\!] = \{\langle [\![\text{hot}]\!], [\![\text{cold}]\!] \rangle, \langle [\![\text{alive}]\!], [\![\text{dead}]\!] \rangle, \langle [\![\text{train}]\!], [\![\text{arrived}]\!] \rangle\}$$

The quantifier *no* will always contains pairs of **antonyms** (e.g. words with inherently opposite meanings)

More quantifiers

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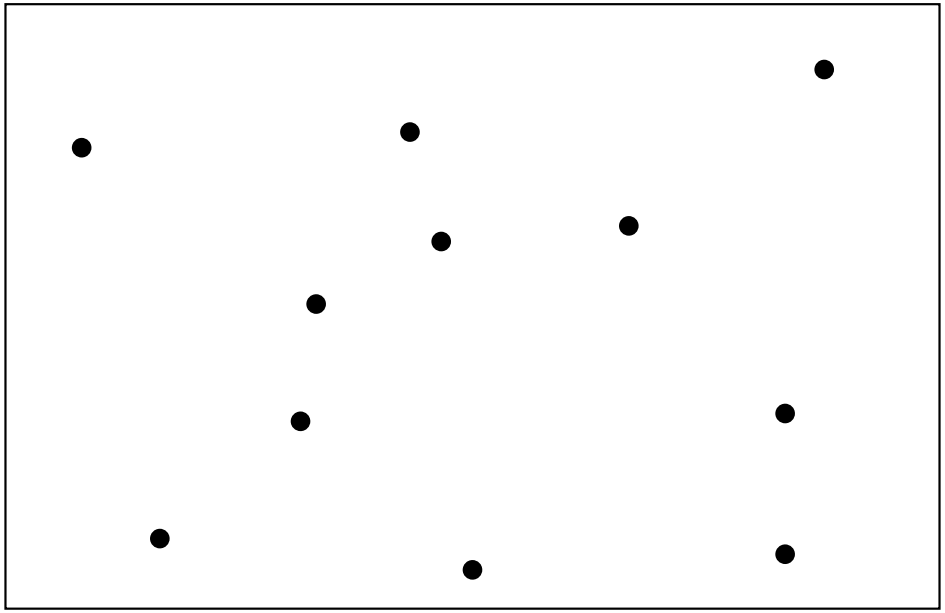

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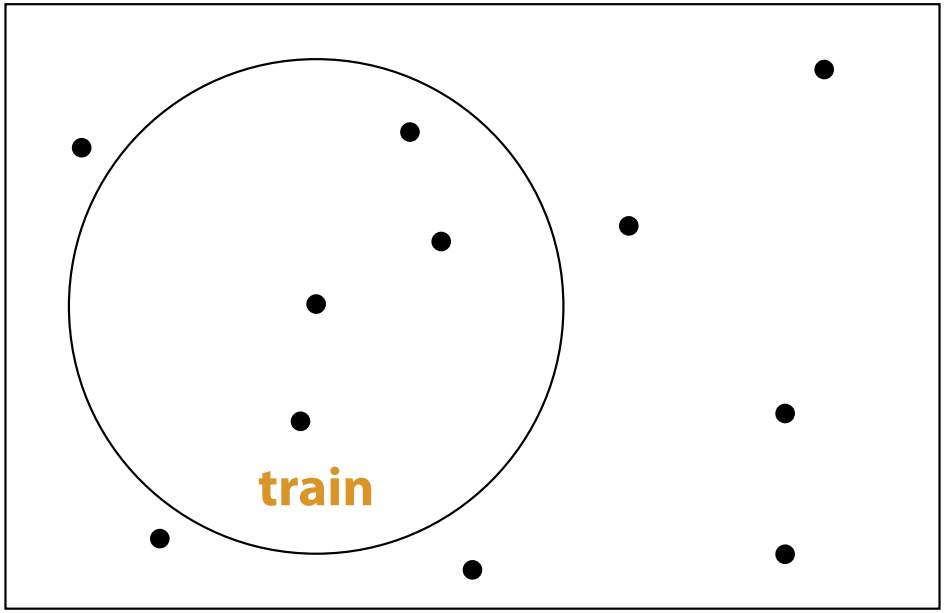

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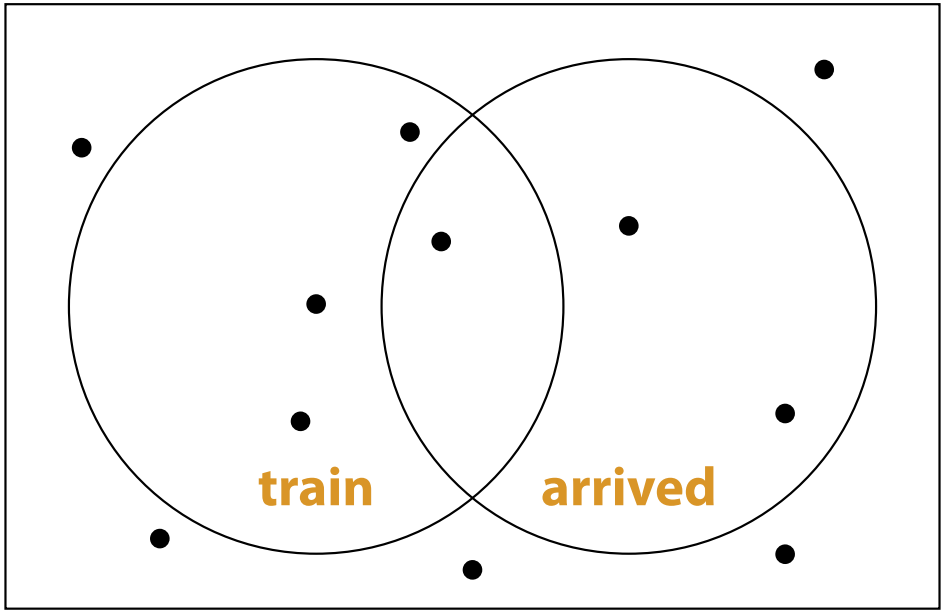

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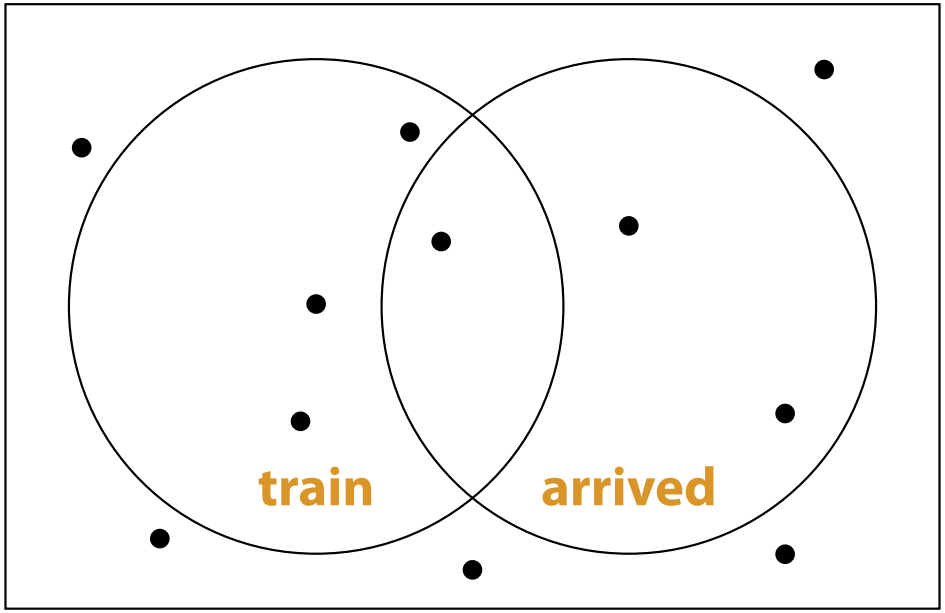

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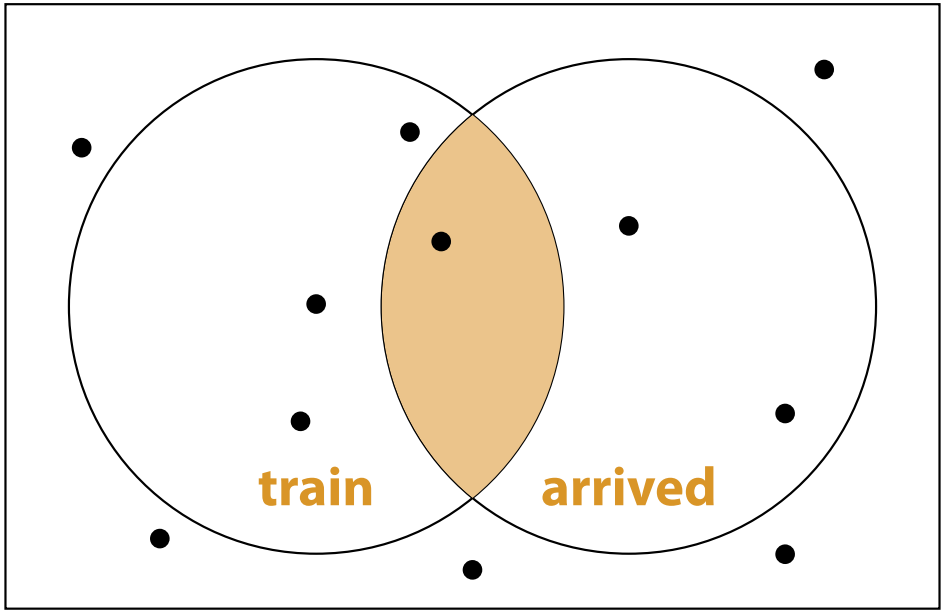

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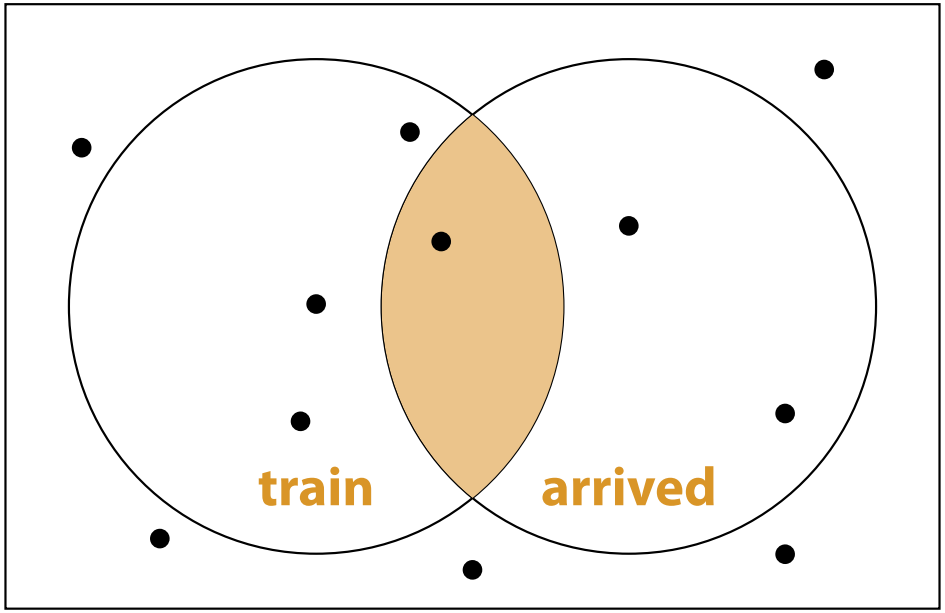

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$$[\![\text{A train arrived}]\!] = \text{true if } \{x : x \text{ is a train}\} \cap \{y : y \text{ arrived}\} \neq \emptyset$$

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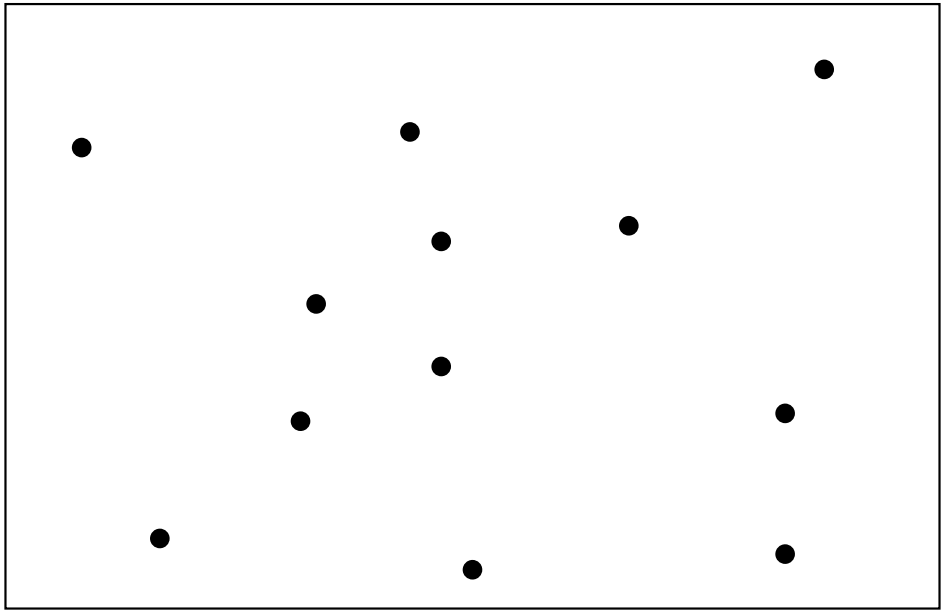

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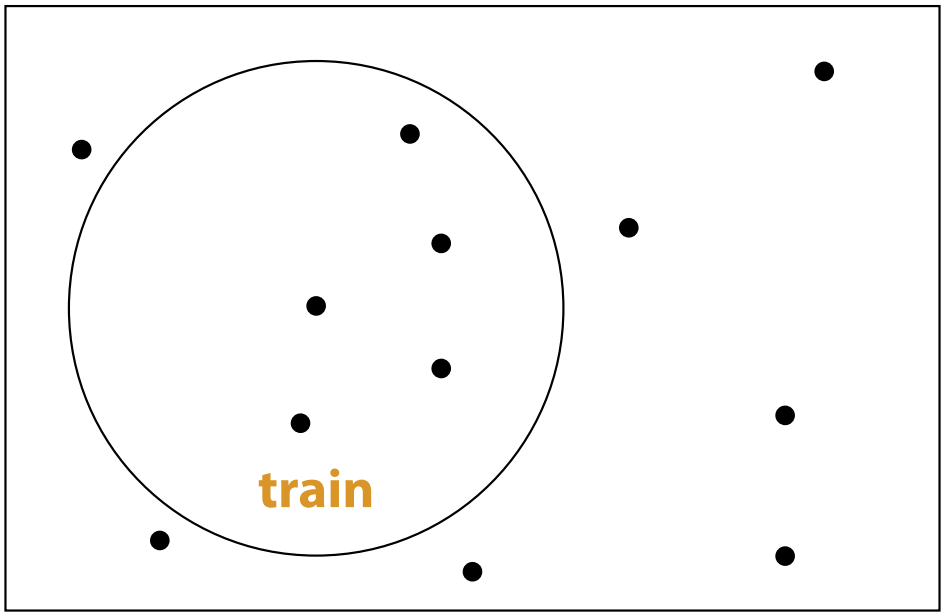

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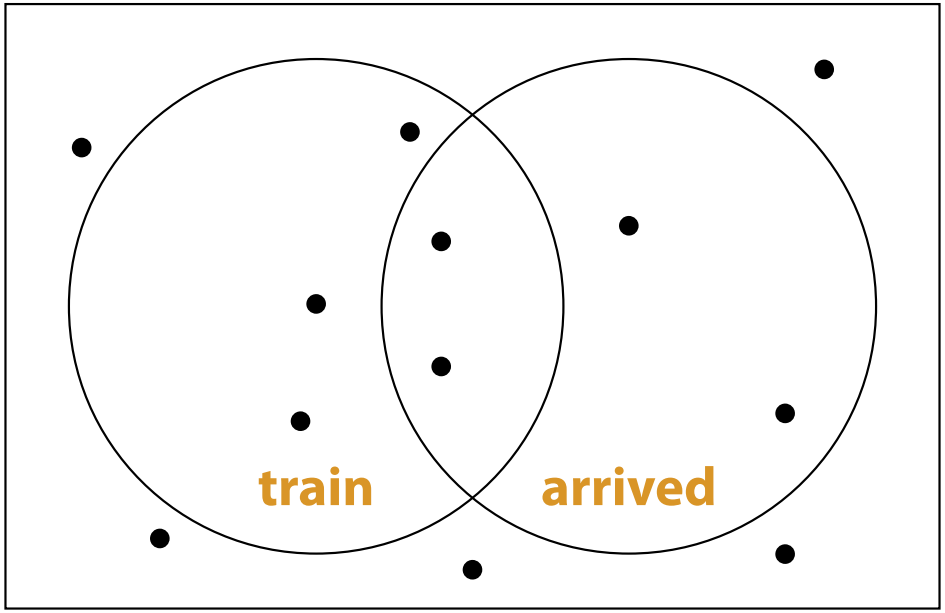

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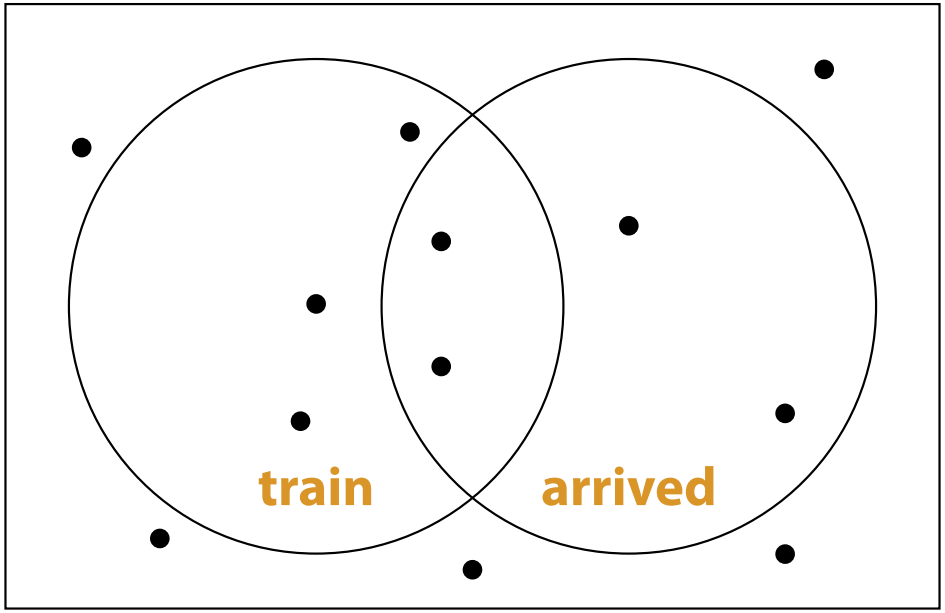

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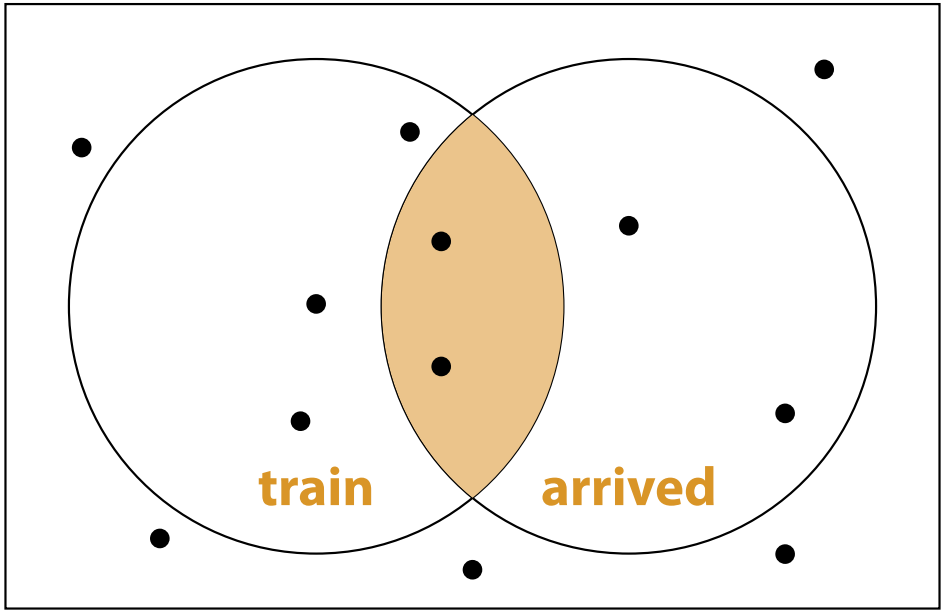

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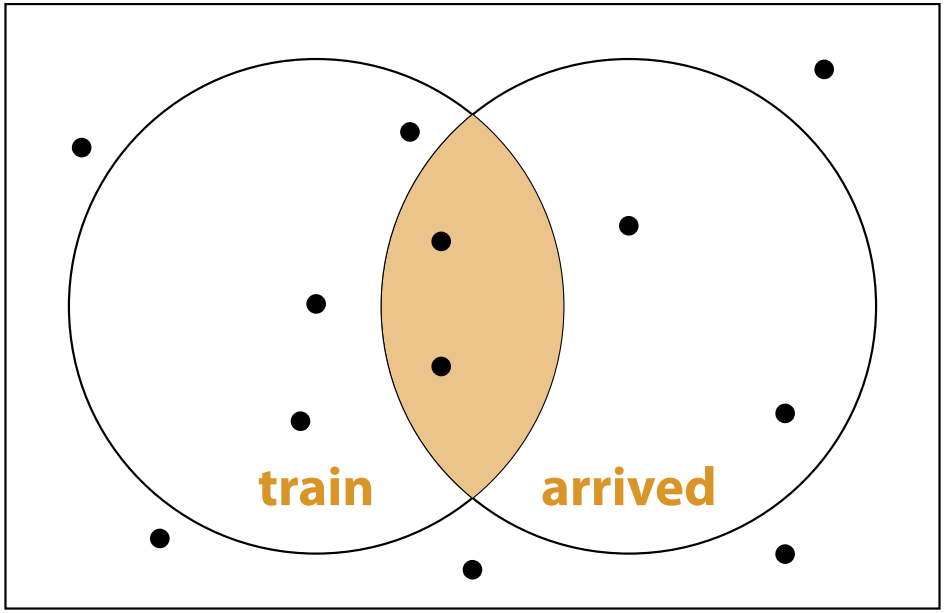

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$$[\![\text{Two trains arrived}]\!] = \text{true if } |\{x : x \text{ is a train}\} \cap \{y : y \text{ arrived}\}| \geq 2$$

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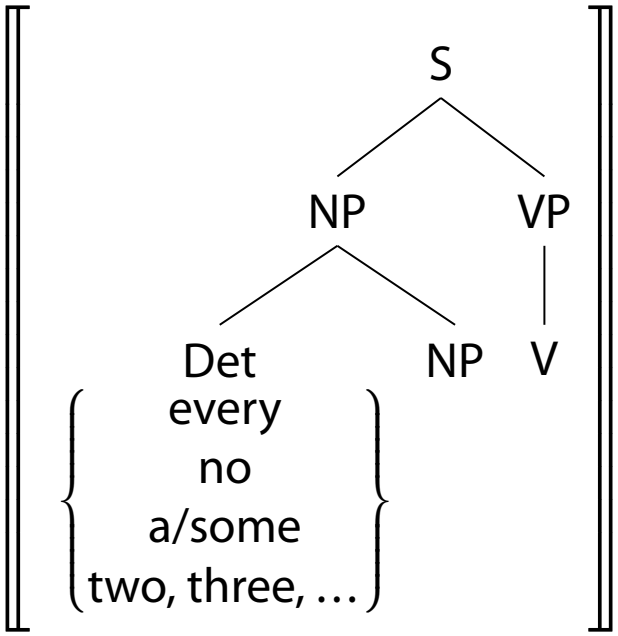

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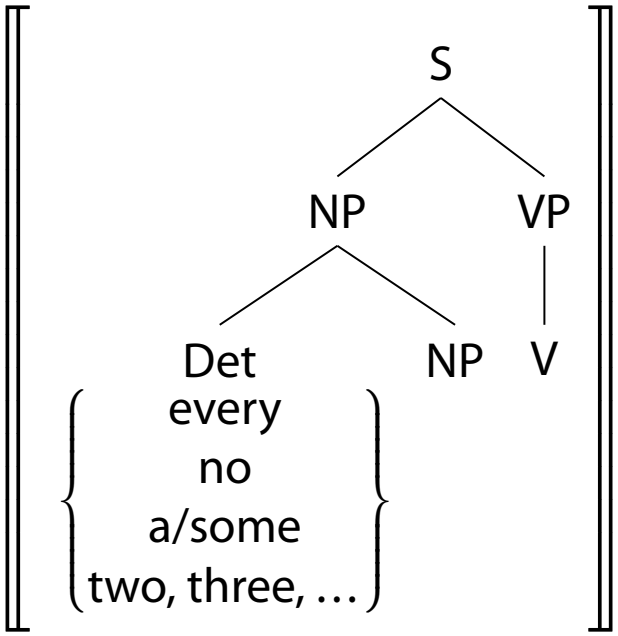

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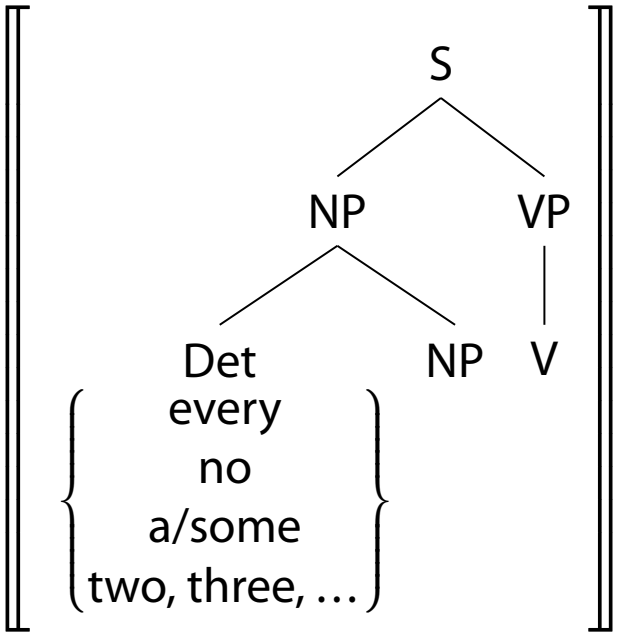

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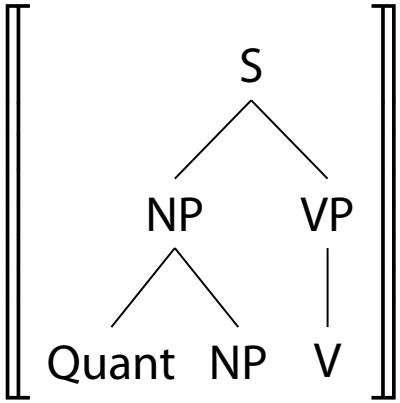

$$= \quad \text{true if } \langle [\![\text{NP}]\!], [\![\text{VP}]\!] \rangle \subseteq [\![\text{Quant}]\!]$$

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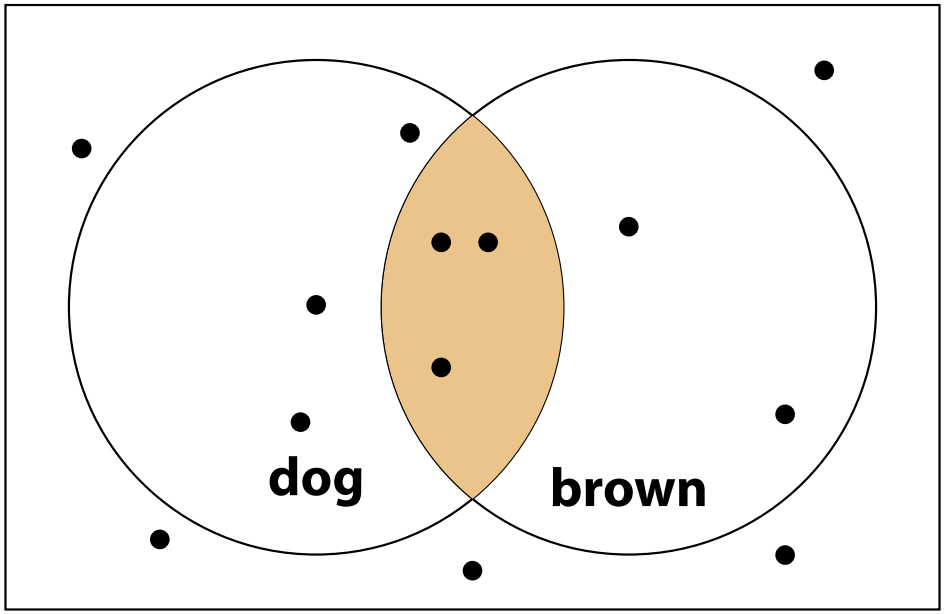
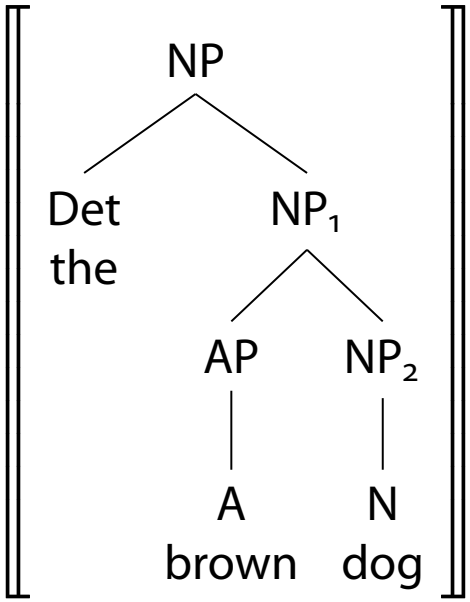


$$[\![\text{brown dog}]\!] = \{x : x \text{ is brown}\} \cap \{y : y \text{ is dog}\}$$

Modification

What about *the brown dog*?



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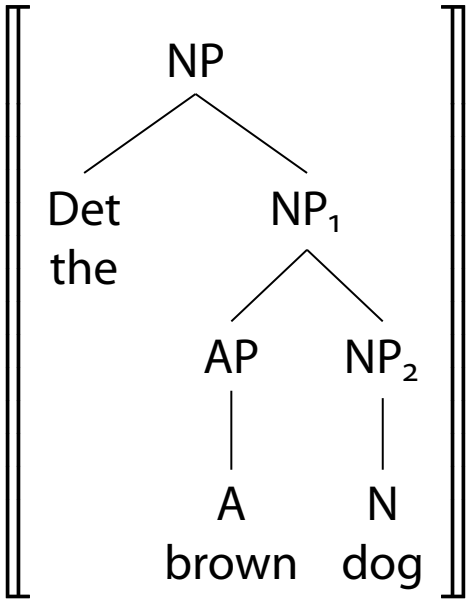

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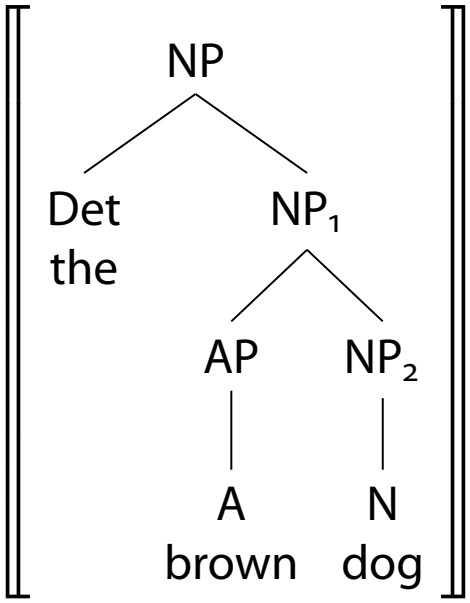

$$= \begin{array}{l} \text{the unique } x \text{ s.t. } x \in [\![\text{NP}_1]\!] \\ \text{the unique } x \text{ s.t. } x \in [\![\text{AP}]\!] \cap [\![\text{NP}_2]\!] \\ \text{the unique } x \text{ s.t. } x \in [\![\text{brown}]\!] \cap [\![\text{dog}]\!] \end{array}$$

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$$\left[\!\left[\begin{array}{c} \text{NP} \\ \text{Det} \quad \text{NP}_1 \\ \text{the} \\ \text{AP} \quad \text{NP}_2 \\ \text{A} \quad \text{N} \\ \text{brown} \quad \text{dog} \end{array} \right]\!\right] =$$

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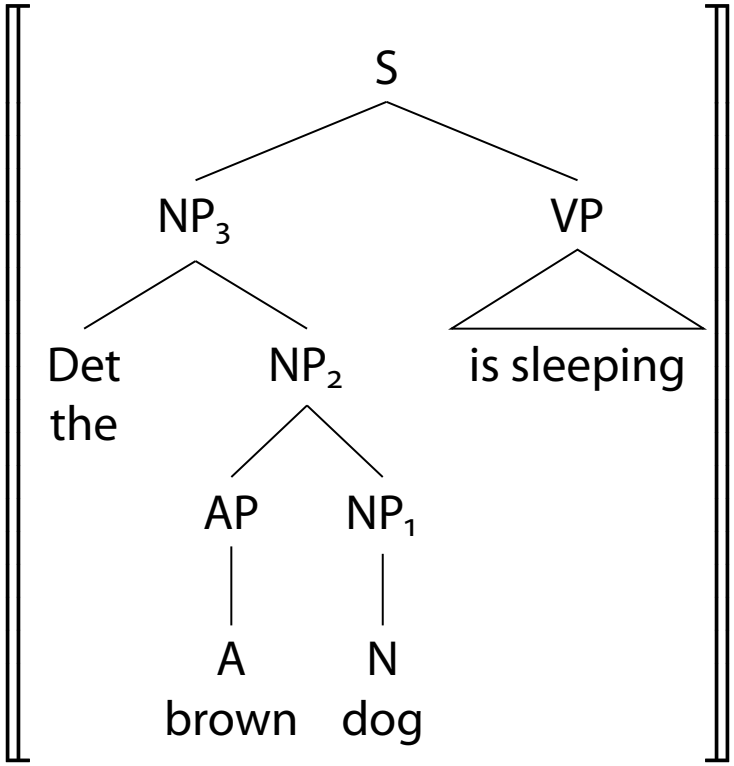

$= \quad \text{true} = [\![NP_3]\!] \in [\![VP]\!]$

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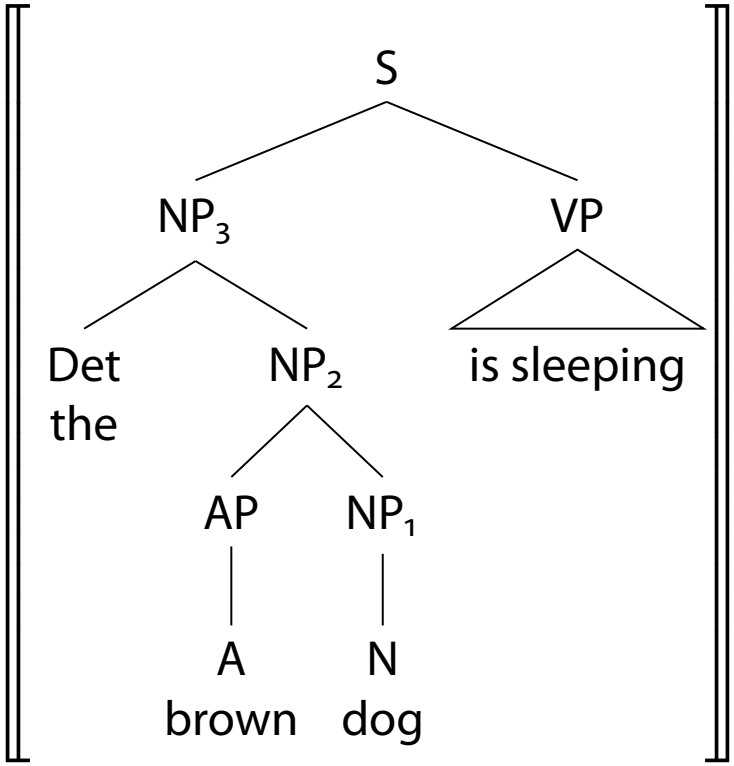
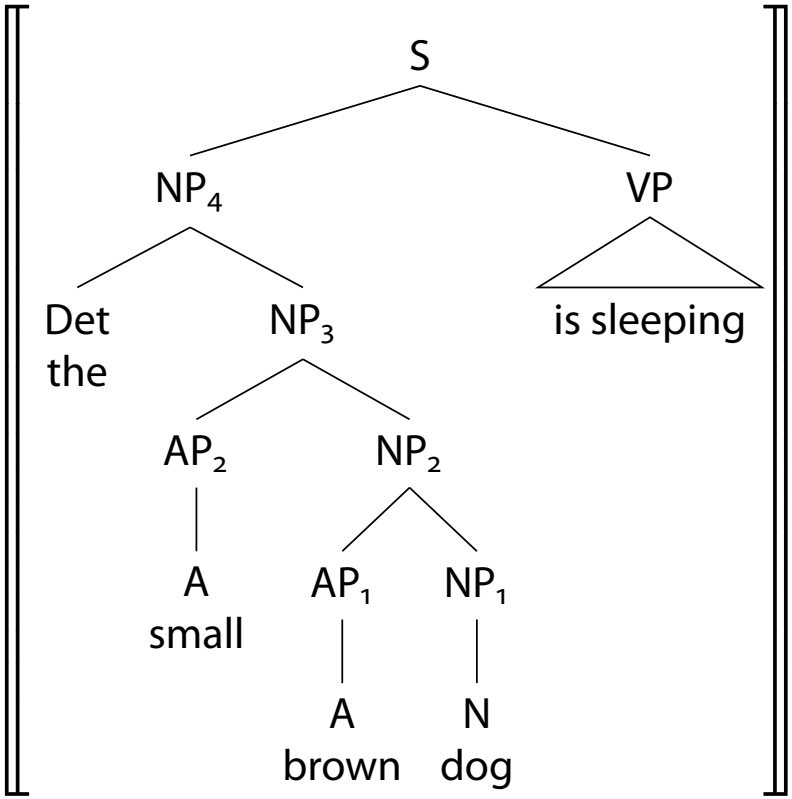


$$= \quad \text{true} = [\![\text{NP}_3]\!] \in [\![\text{VP}]\!]$$

$$\frac{\text{the unique } x \text{ s.t. } x \in \{y : y \text{ is brown}\}}{\underline{\cap \{z : z \text{ is dog}\}} \in \{q : q \text{ is sleeping}\}}$$

Modification

What about multiple adjectives like with *the small brown dog is sleeping*?



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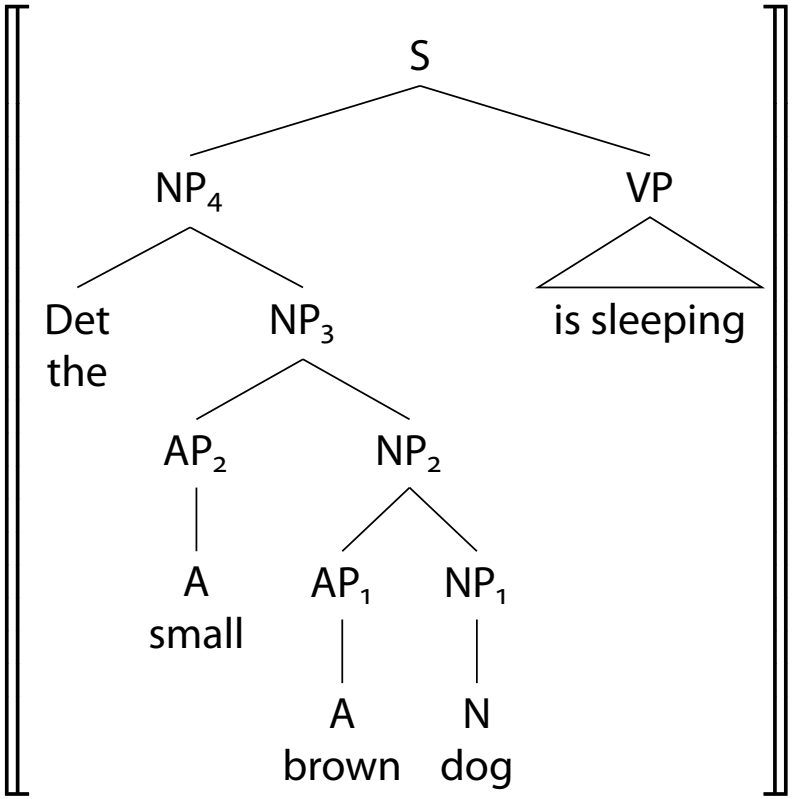

=

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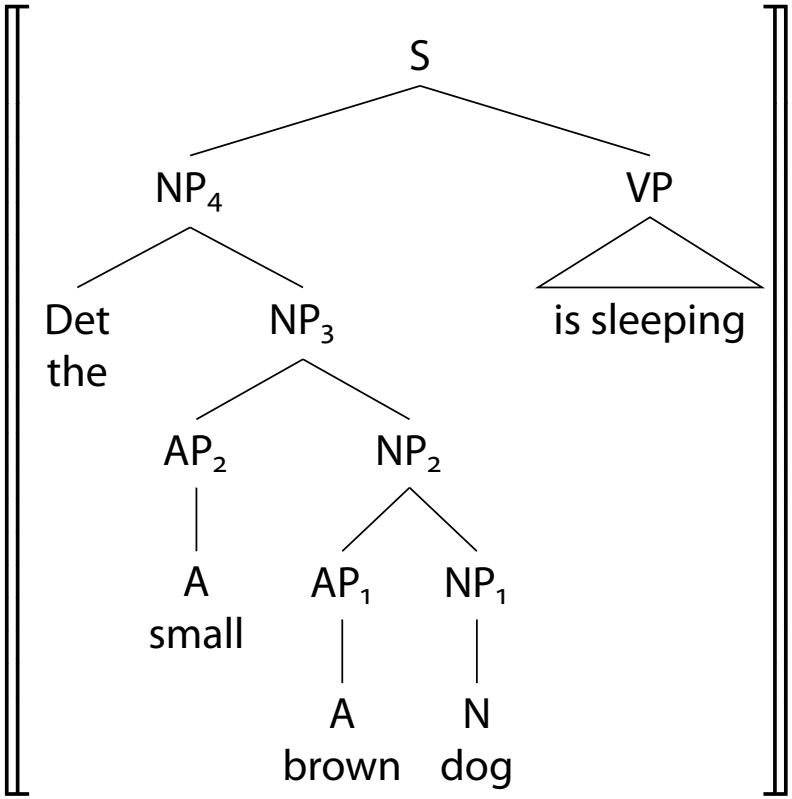

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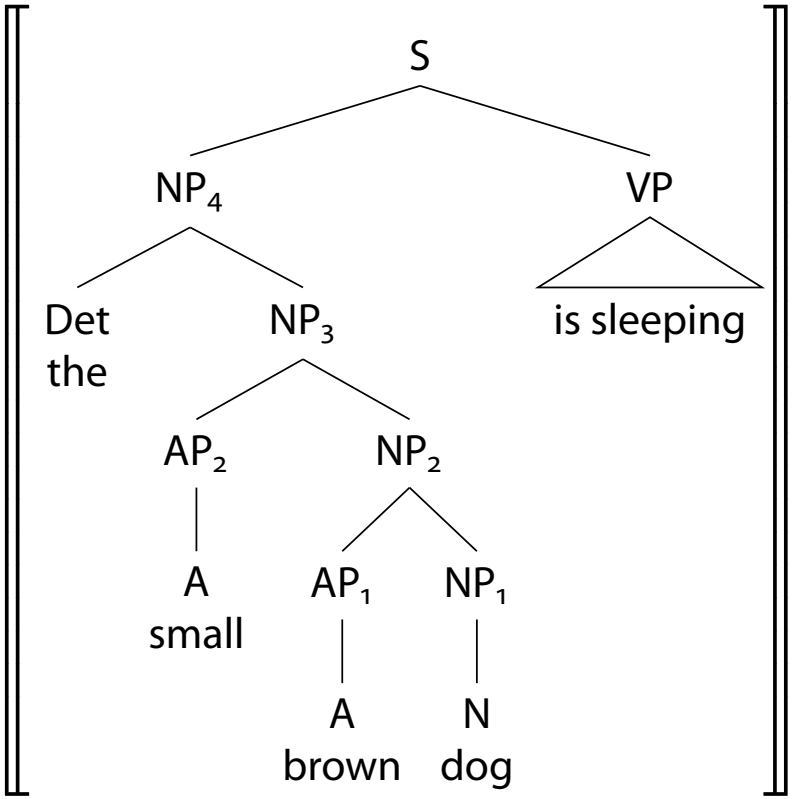

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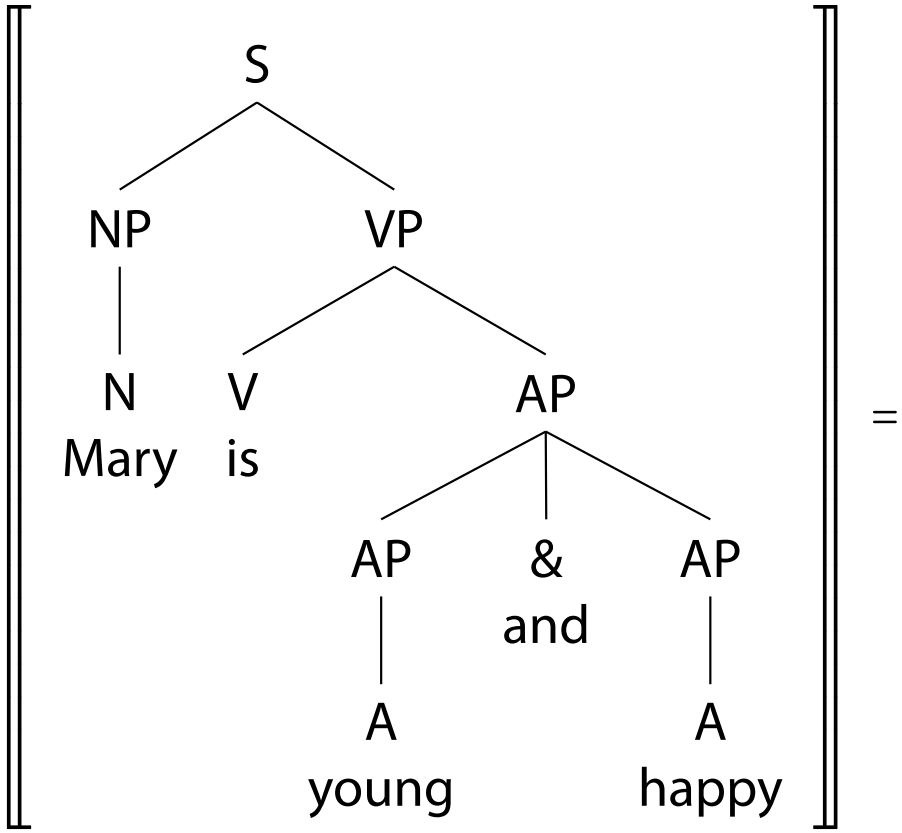
Coordination

What is the meaning of coordinated adjectives like *young and happy*?



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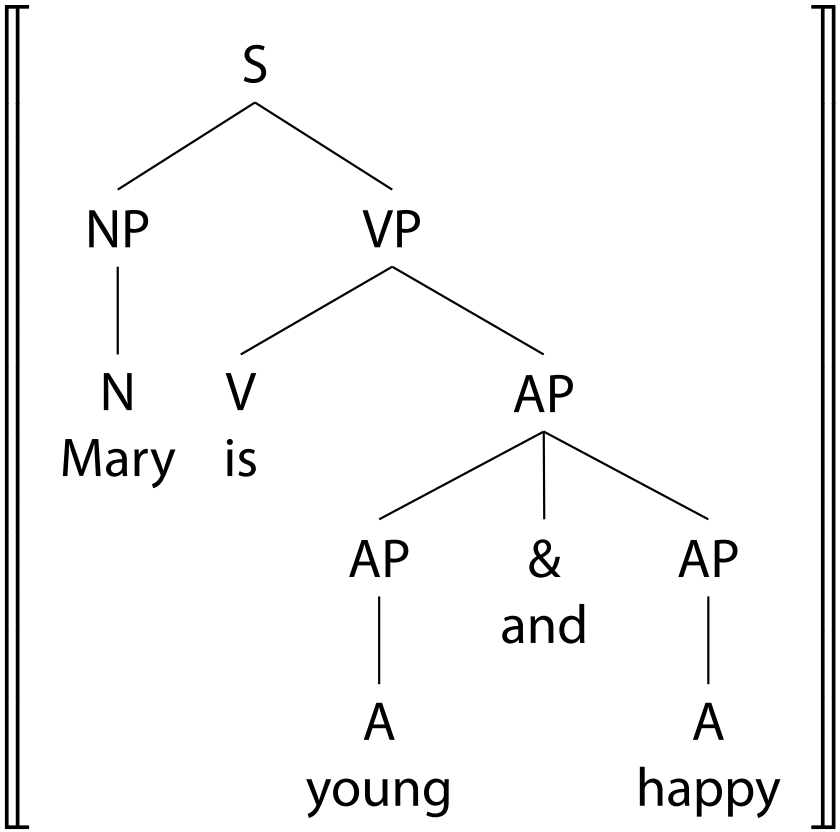

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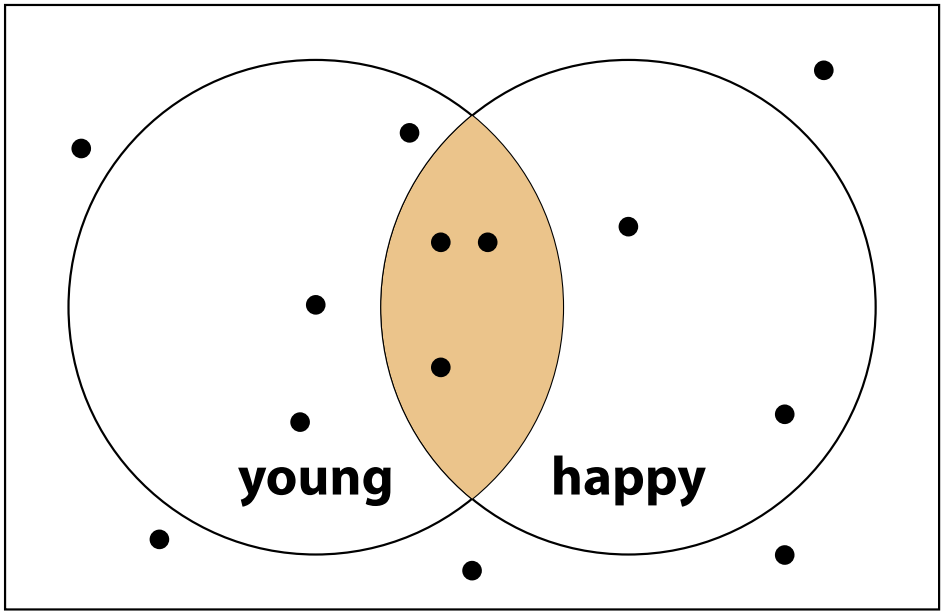

Mary $\in \{x : x \text{ is young}\} \cap \{y : y \text{ is happy}\}$

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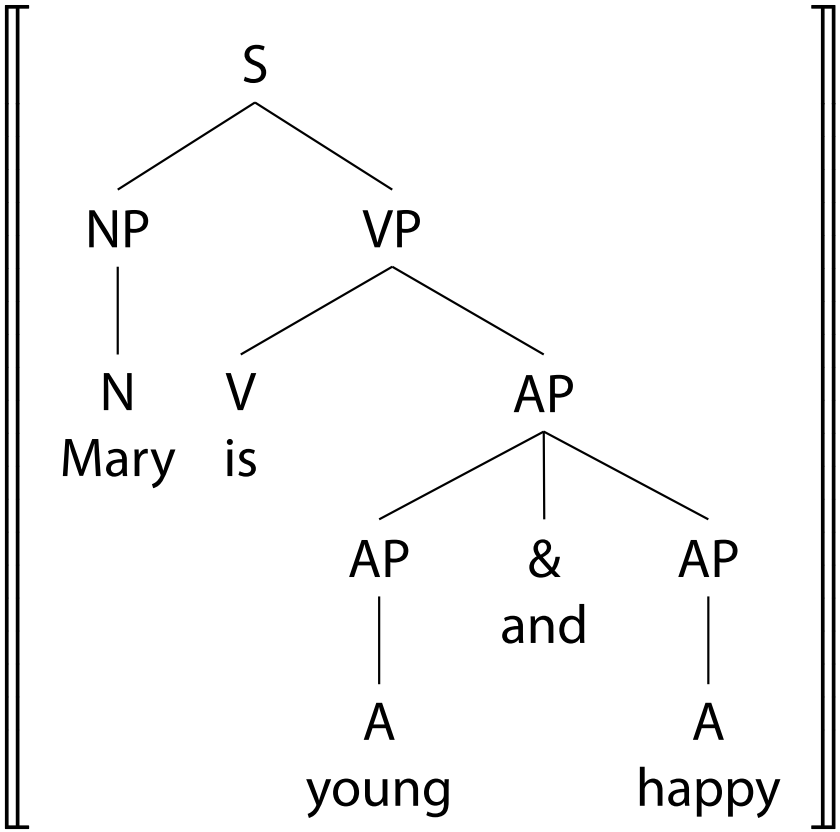

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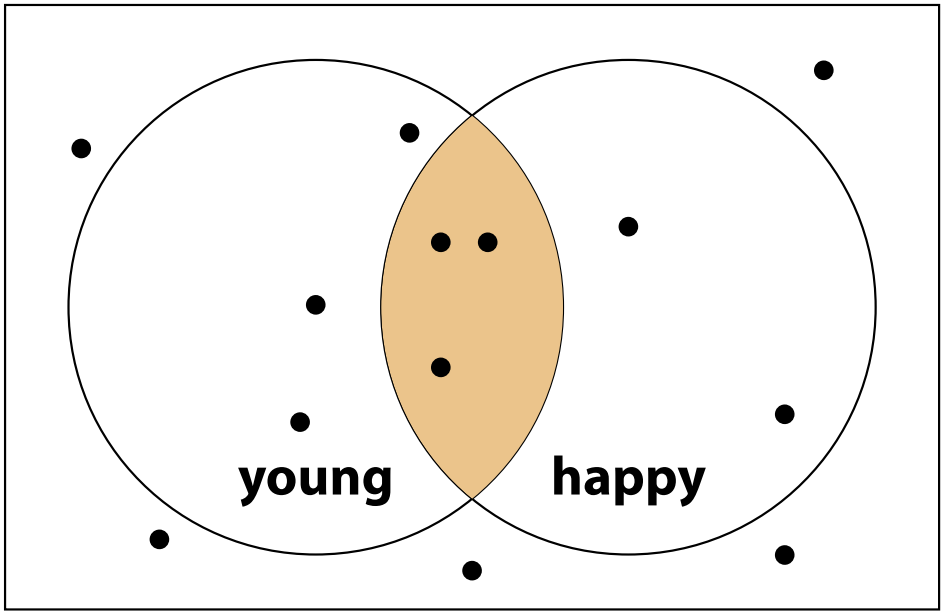
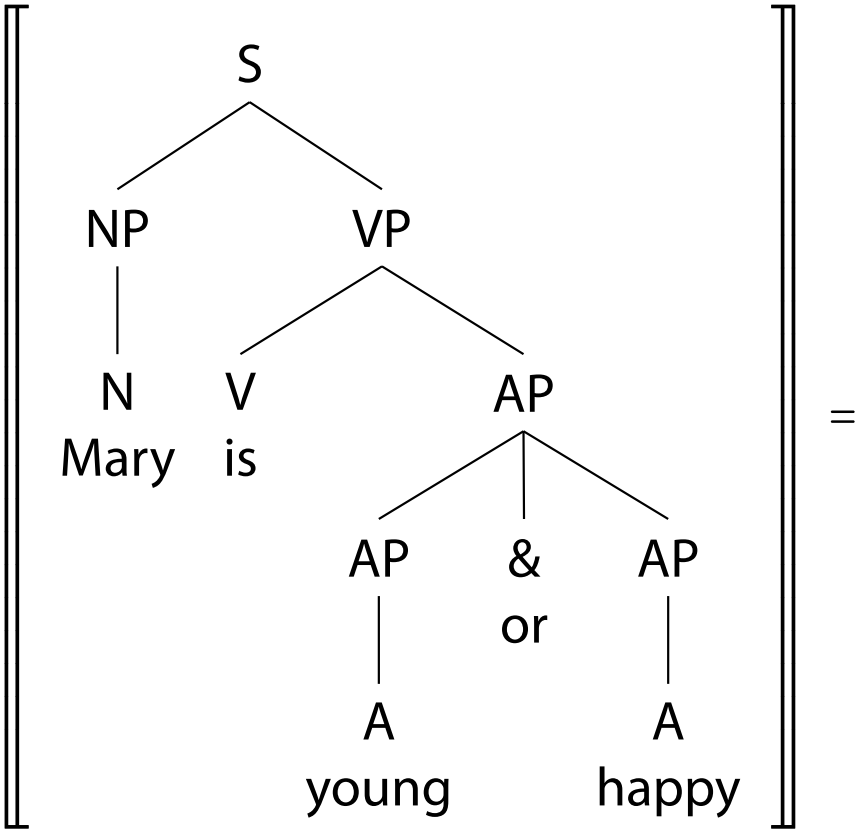


Mary $\in \{x : x \text{ is young}\} \cap \{y : y \text{ is happy}\}$

It is just set intersection again!

Coordination

What is the meaning of coordinated adjectives like *young or sad*?



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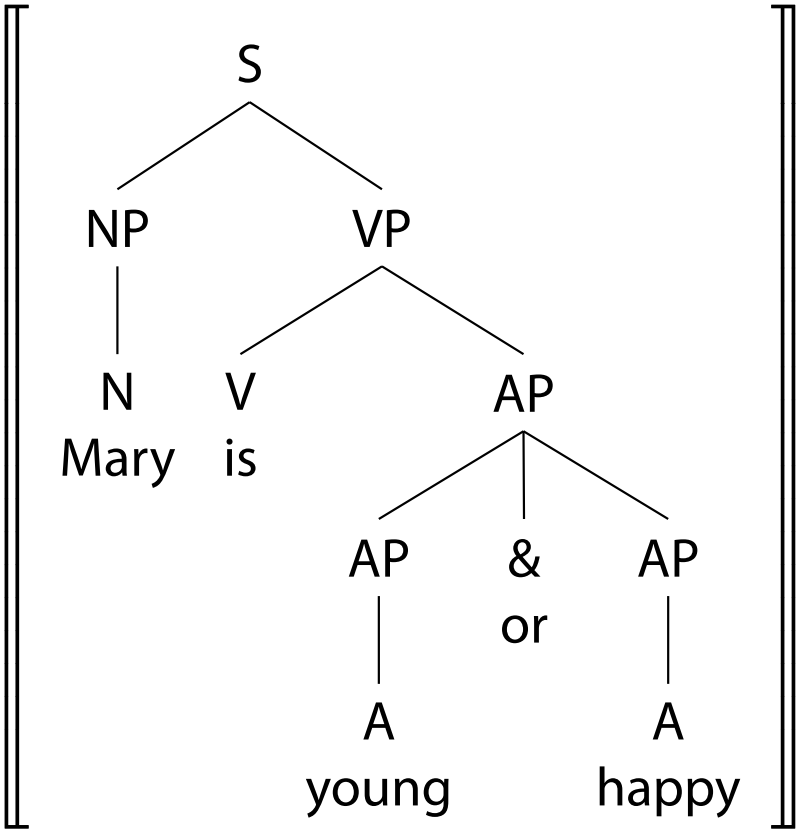

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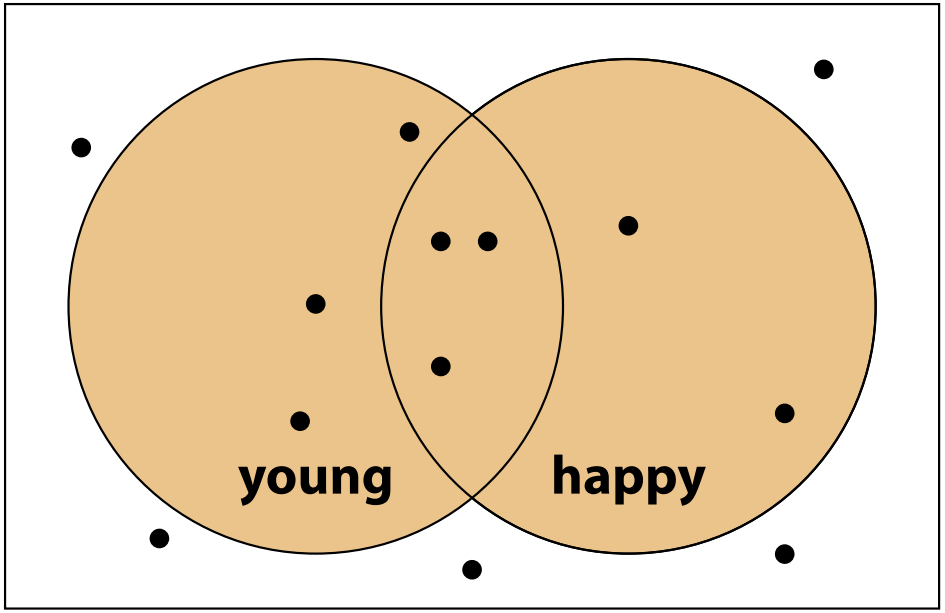

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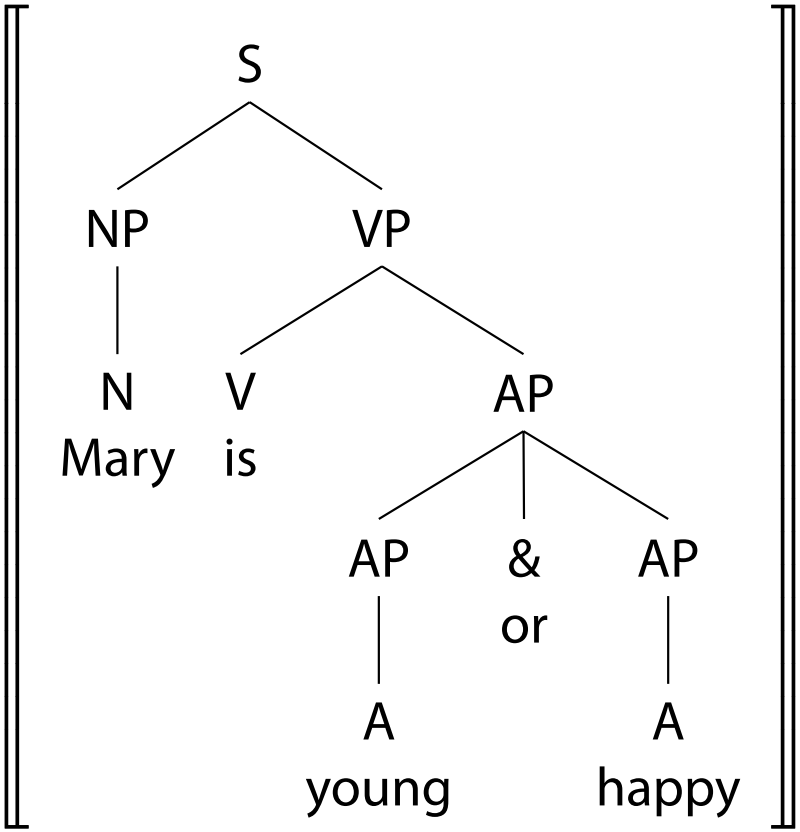

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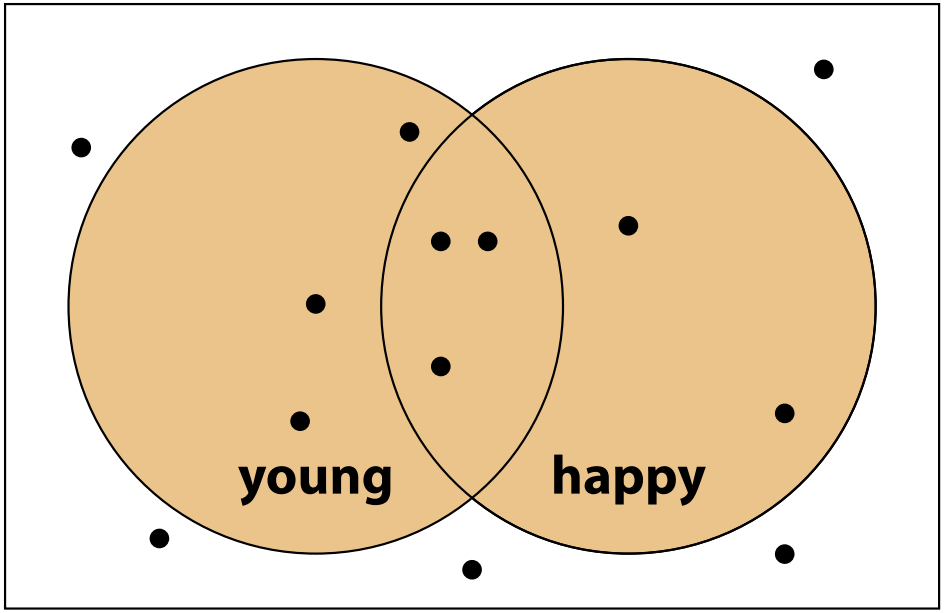

Mary $\in \{x : x \text{ is young}\} \cup \{y : y \text{ is happy}\}$

The coordination *or* involves the **union** ($\cup$) of two sets!

Negation

What about negation in phrases like *not happy*?

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It involves the **complement** of some set {a, b, c} ($\overline{\{a, b, c\}}$), i.e. the set of elements that are **not** in a given set.

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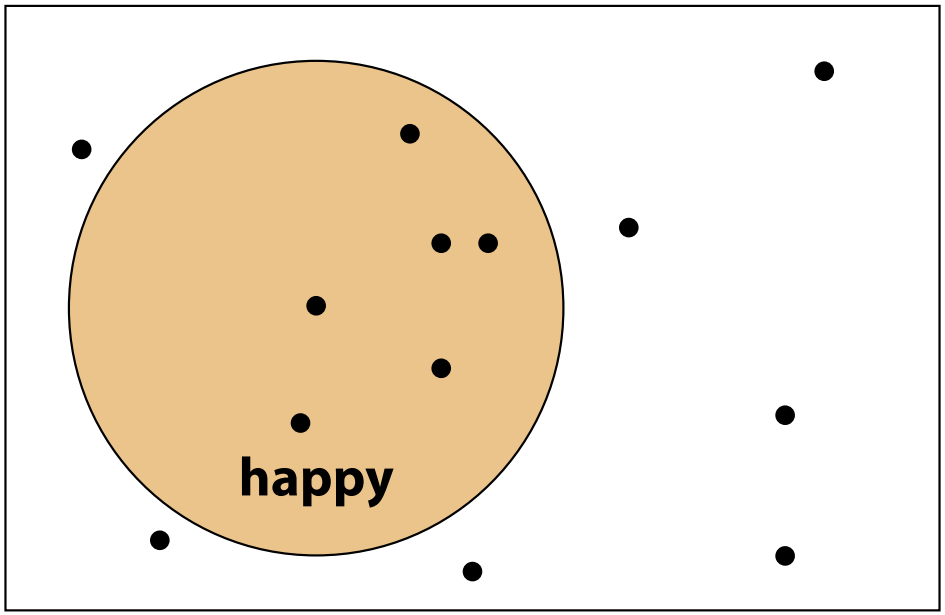

$[\![\text{Mary is happy}]\!]$ = true if Mary $\in \{x : x \text{ is happy}\}$

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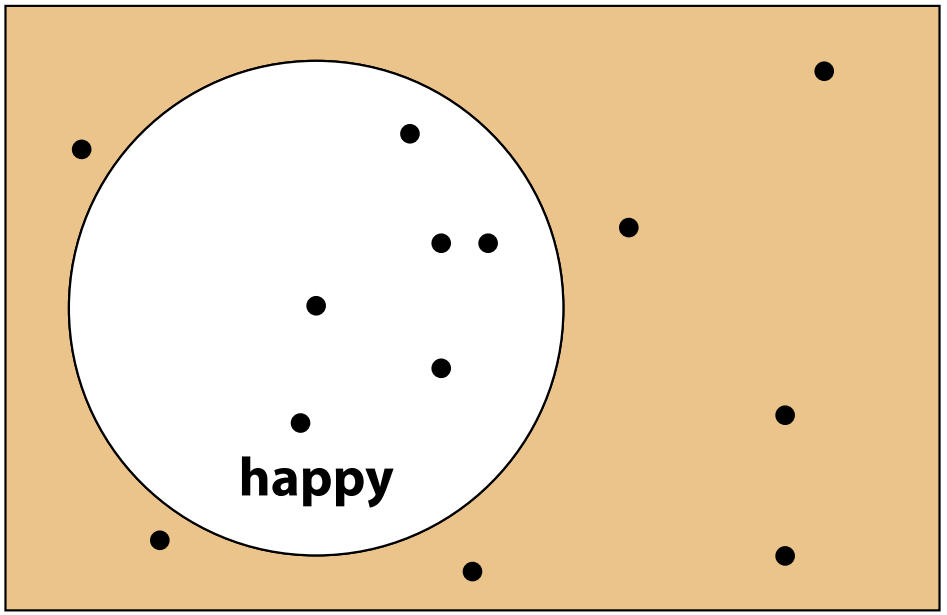
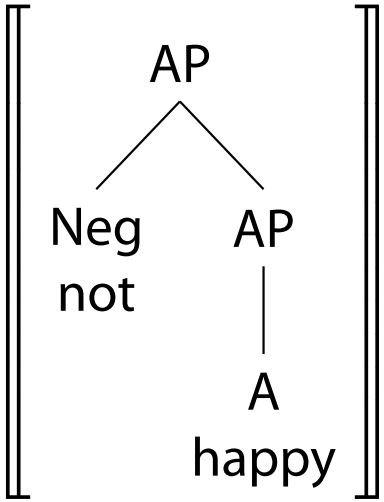


$[\![\text{Mary is not happy}]\!] = \text{true if Mary} \in \overline{\{x : x \text{ is happy}\}}$

Negation



$$[\![\text{not AP}]\!] = \overline{[\![\text{AP}]\!]}$$

$$[\![\text{not AP}]\!] = \overline{\{x : x \text{ is happy}\}}$$

Ambiguity

What are the two meanings of *Peter is not tall and thin*?

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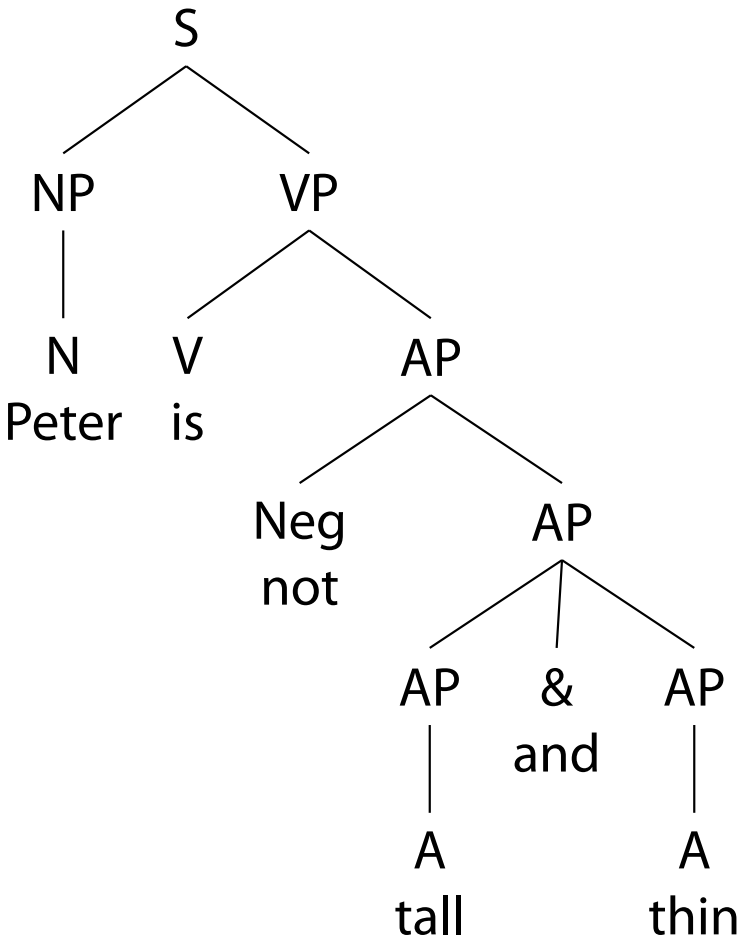

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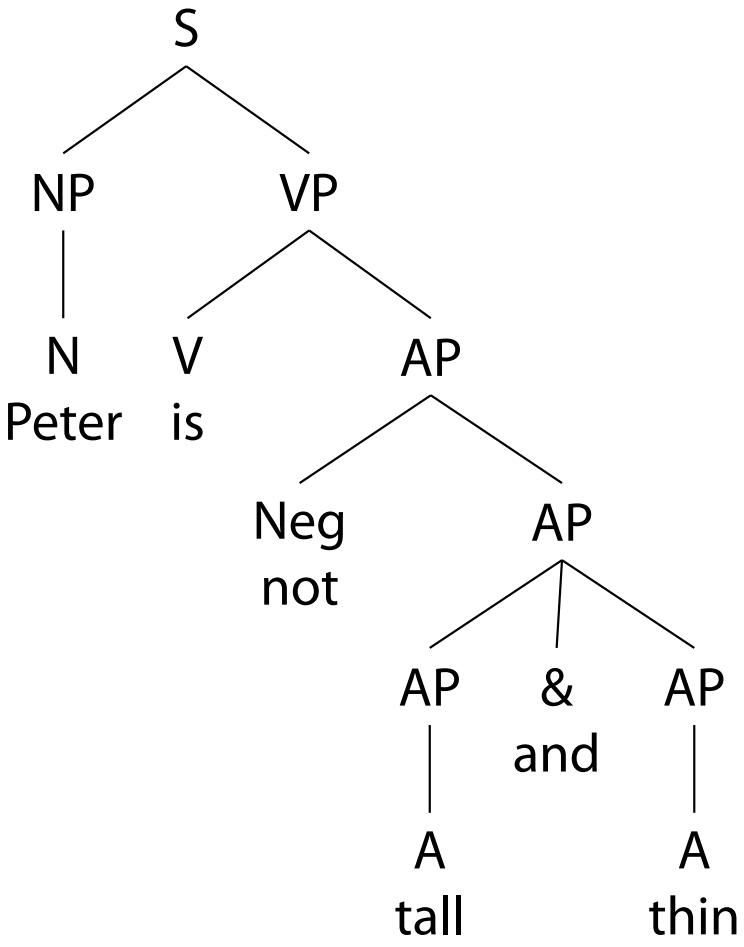

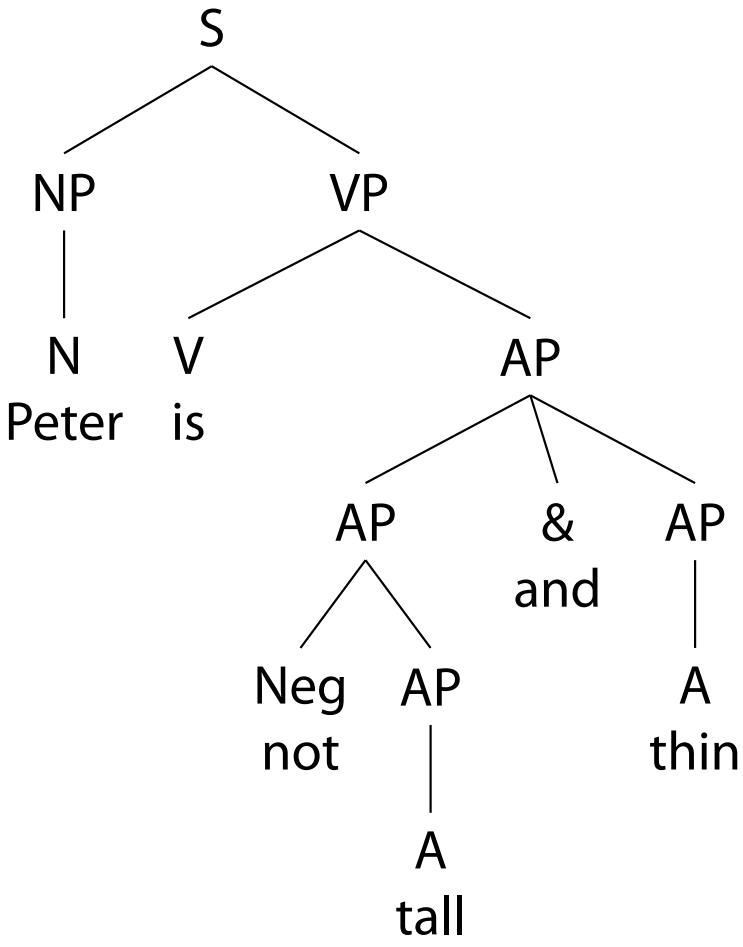

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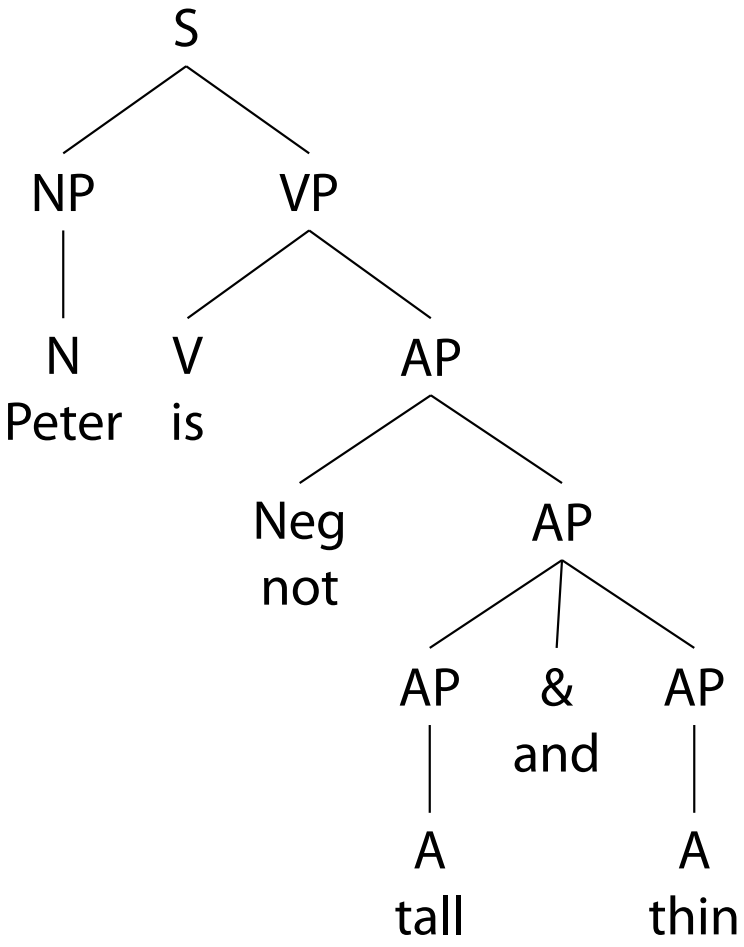

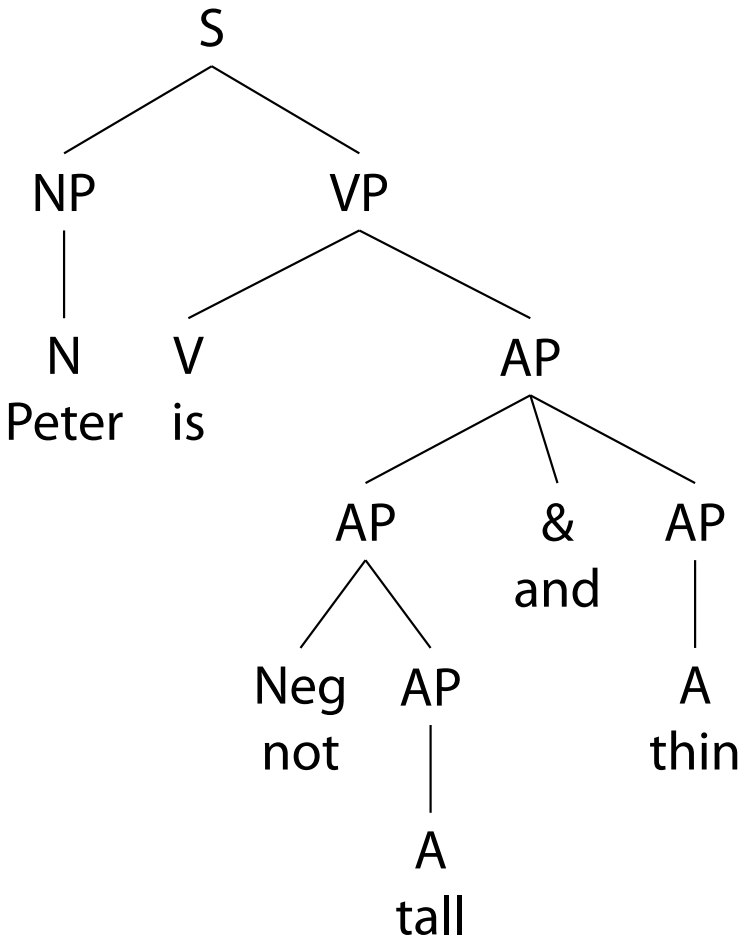

This is a structural ambiguity!

Ambiguity

Peter is [not tall and thin]

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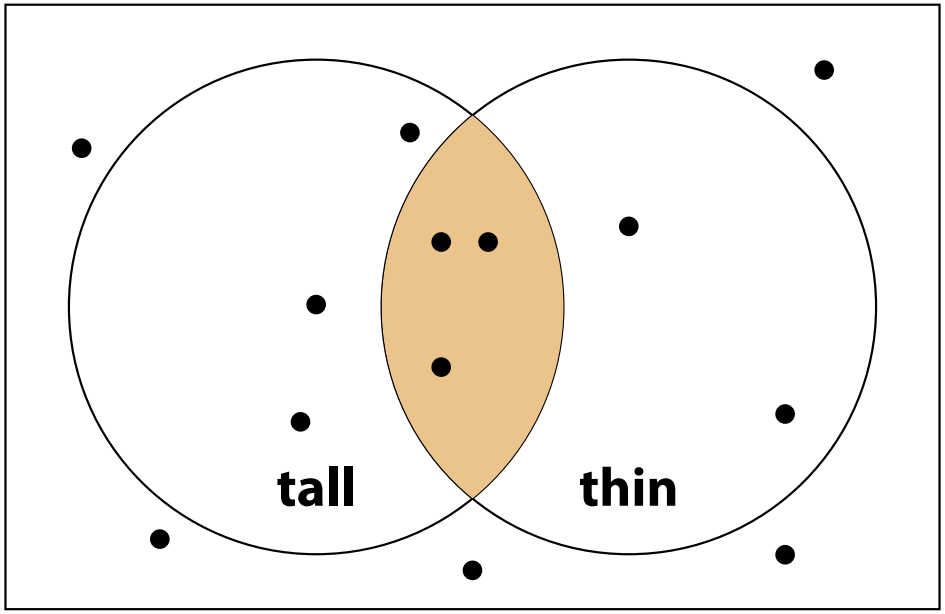
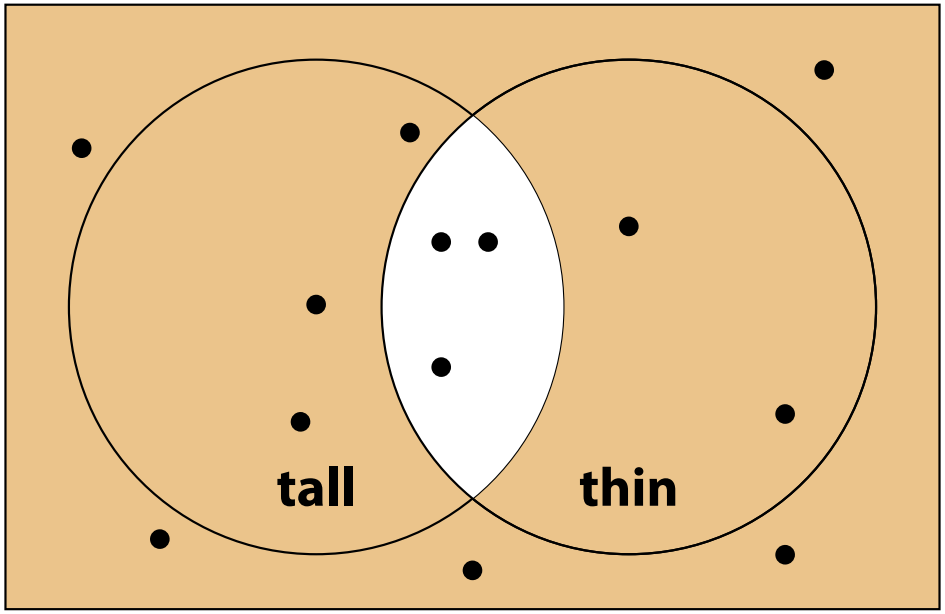
Peter is [not tall and thin]



$[\![\text{tall and thin}]\!] = \{x : x \text{ is tall}\} \cap \{y : y \text{ is thin}\}$

Ambiguity

Peter is [not tall and thin]



$$[\![\text{not [tall and thin]}]\!] = \overline{\{x : x \text{ is tall}\} \cap \{y : y \text{ is thin}\}}$$

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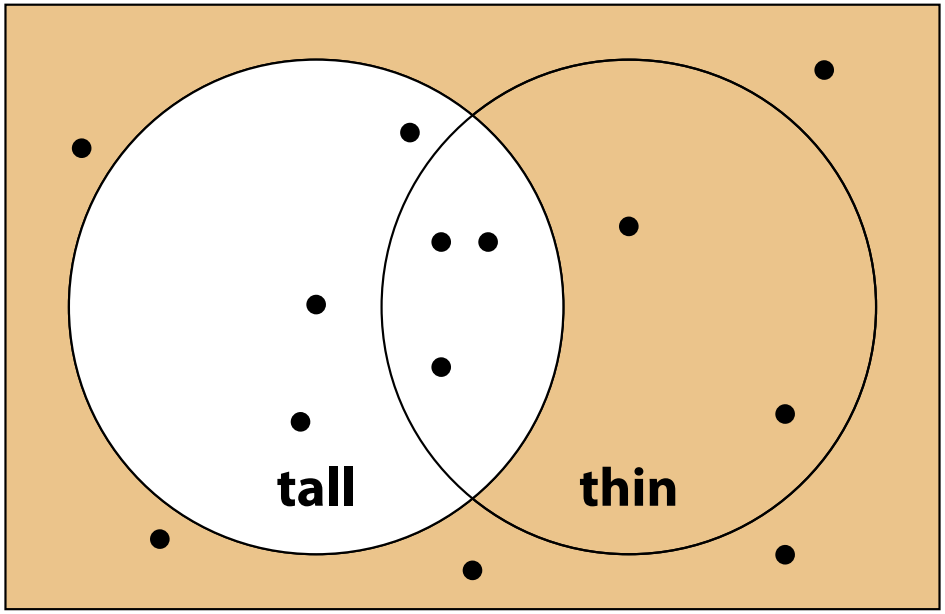

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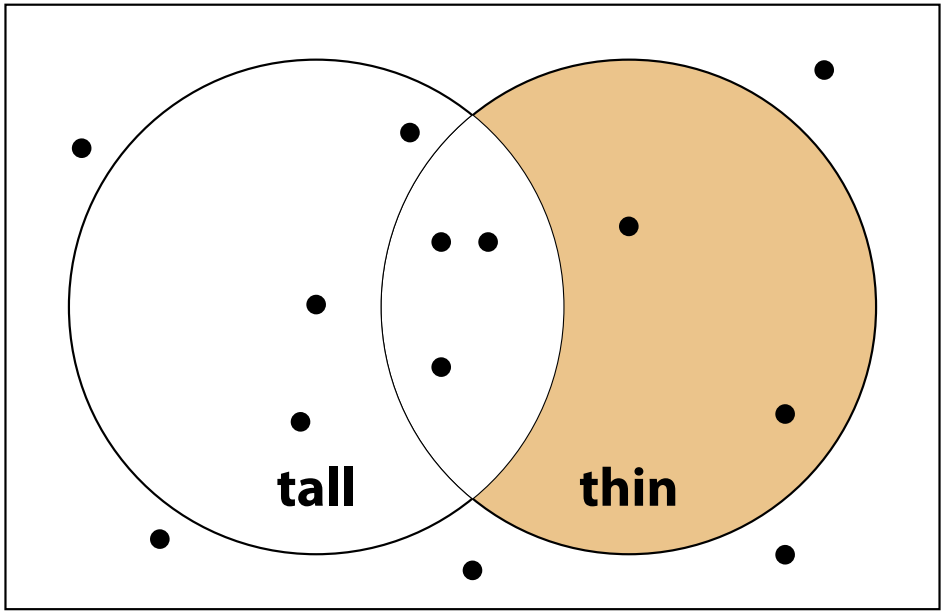

$$[\![\text{[not tall] and thin}]\!] = \overline{\{x : x \text{ is tall}\}} \cap \{y : y \text{ is thin}\}$$

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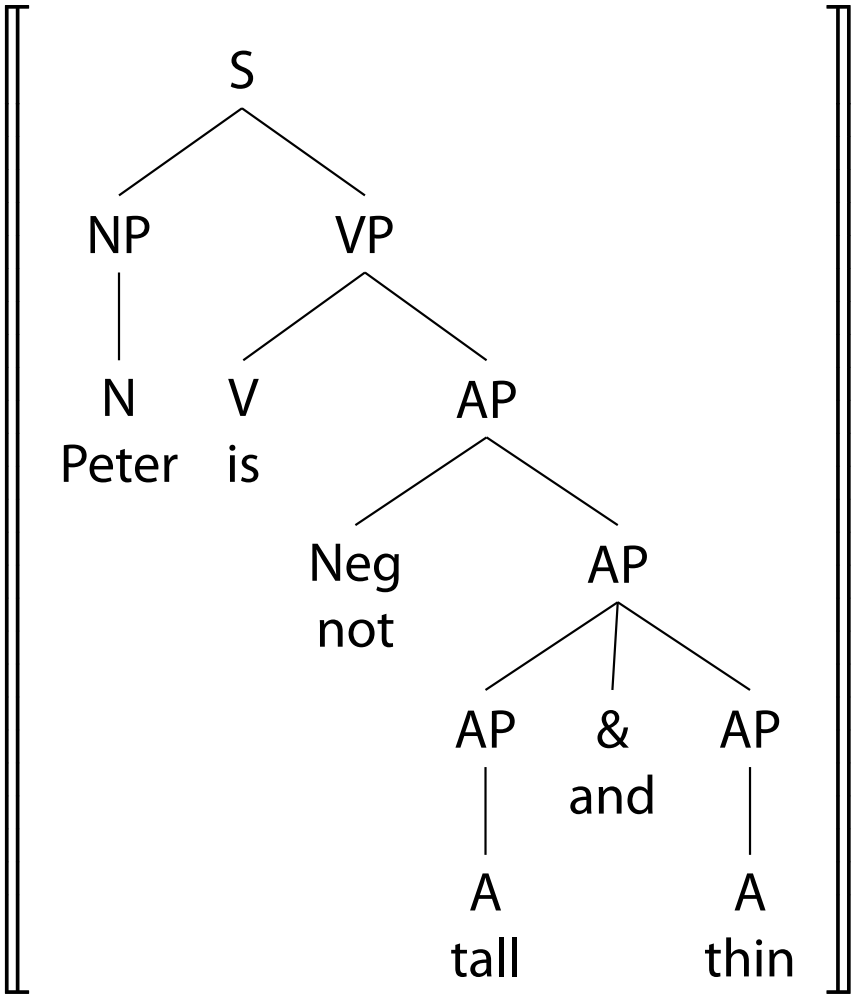

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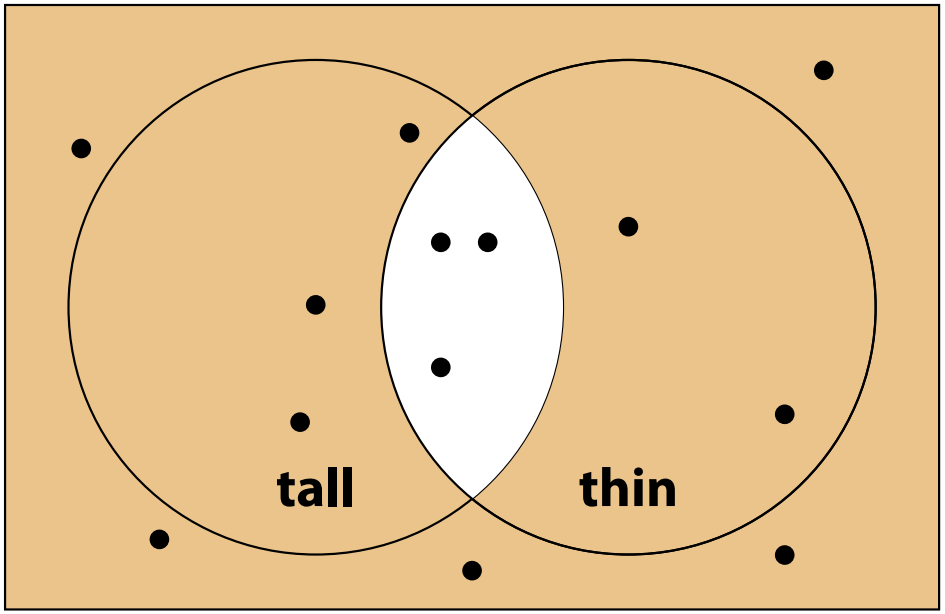
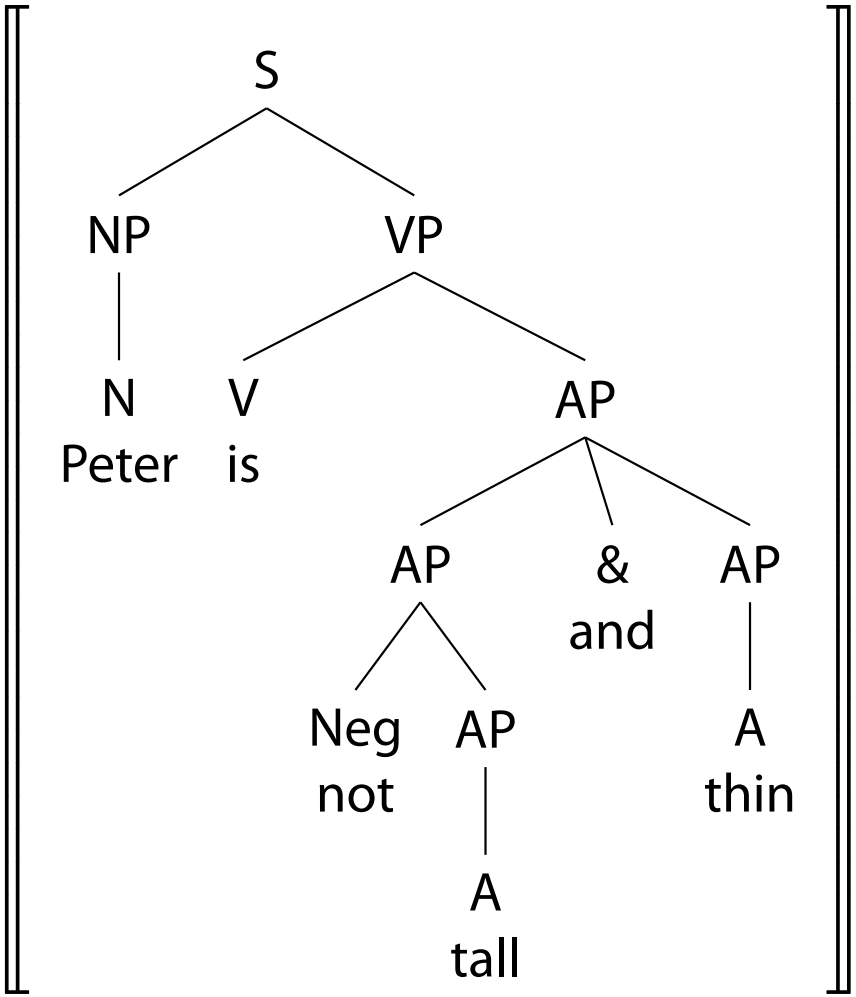


true if Peter $\in \overline{\{x : x \text{ is tall}\} \cap \{y : y \text{ is thin}\}}$

Ambiguity



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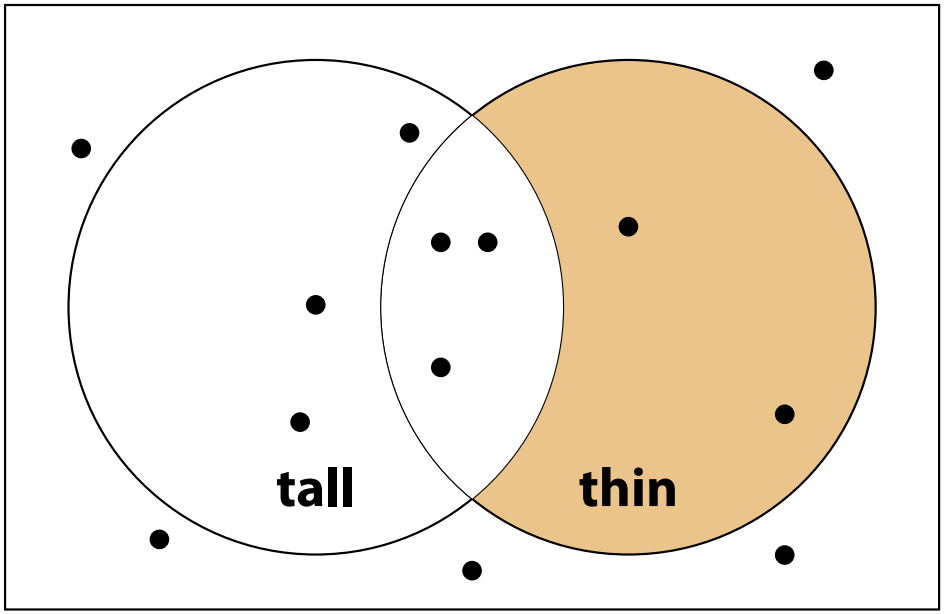

true if Peter $\in \overline{\{x : x \text{ is tall}\}} \cap \{y : y \text{ is thin}\}$